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ON THE TEACHING OF NATURAL SCIENCE IN THE SECONDARY SCHOOLS.[1]

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The "higher schools" (Gymnasia, Realgymnasia and Oberrealschulen) must give to the growing youth a mass of information and a variety of skills that are to serve as a *general* foundation for his later activities. They are not intended to prepare for any *special* vocation.

The carrying out of this aim of the high schools offers a difficulty at the very start, in the choice of the educative material. There is abundance of valuable material well adapted to serve as a basis for general training for life. But it is impossible to teach everything: our times already suffer from an almost morbid tendency to accumulate as much knowledge as possible, and to expect this knowledge of the educated man. When we consider that the excessive accumulation of facts in the brain but hinders the development of the logical processes and deeper mental activity, we should be more likely to take thought how to reduce the amount of knowledge in our education.

The material must then be selected; and this selection may be made from different points of view.

We may select the material according to the inclinations of the pupil. Since the pupils have varying capacities, one more for languages, another more for mathematics, etc., this fact may serve to differentiate the higher schools according to the subjects taught. This has indeed already taken place to a certain extent for the different types of high schools. As against the schools that attempt to mold all the students after one pattern, I consider schools differentiated according to this point of view

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[1](Translation and abstract from "Beiträge zur Frage des naturwissenschaftlichen Unterrichtes an den höheren Schulen," Jena, 1904, by Benj. C. Gruenberg, DeWitt Clinton High School, New York.)

Note: While the references to school conditions are the outcome of experience with German education, many of the points will apply equally well to conditions in this country.

decidedly worth striving for. But under all circumstances, every variety of high school, without exception, must be required to justify itself with respect to the later activities of the pupils.

A second consideration in the selection of the material is also very important, and must not be overlooked: that is the selection according to the demands of the contemporary civilization. But the civilization or culture of a people is subject to development, and so the demands that it makes upon the education of the individual as an adjustment to life are also subject to change. It is in this respect that the high schools seem to me to be most backward. This is not the place to discuss the much disputed question of the importance and value of the so-called "humanistic" education for modern civilized life. But I would like to express a view on the educative value of the classic languages so far as to say that the place they occupy in the high school seems to me to be very disproportionate, from the point of view of our modern culture, inasmuch as they take away the air from those knowledges and skills which our present day civilization demands imperatively from every educated member. We are not Greeks and Romans. The civilization of to-day is thoroughly different from that of the classic peoples. It is everywhere characterized by the application of a deeper understanding of nature. The arrangements of our everyday life that surround us at every step demand of us a far-reaching knowledge of animate and inanimate nature. Our technical and hygienic appliances, for example, are unintelligible and cannot be properly used without a mass of natural science—and what is more important still—a certain training in scientific thinking. These factors of culture are, however, of special importance as educative material for all those who will *not* turn to technical, scientific or medical pursuits upon the completion of the school course. The physician, the scientist, the engineer, etc., must acquire the necessary knowledge and skill in the course of his special studies. The jurist, the philologist or the theologian who has not acquired these extremely important elements while at school, will later find no opportunity for getting them, and runs the constant risk of atoning for his shortcomings through unpleasant or even dangerous experiences with his own person. What naive ideas on matters of hygiene, chemistry or physics does one not meet with in the lives of jurists or linguists! There is here evident a serious defect in our choice of the educative material in the schools, and in its application to the needs of

civilized life. *Science teaching in the high schools is demanded not for the training of those who are to enter scientific professions, but for the training of just those who do NOT select a medical or scientific career.* Our civilization develops and changes rapidly; we should seek to keep up with its demands by the careful selection of the subjects taught in the schools.

Laboratory practice courses, conferences and examinations of medical students whose earlier preparation was in the high schools (Gymnasien), have given me ample opportunity to learn the character of the skill and knowledge which such students bring with them from the schools. So far as knowledge is concerned, the small amount of positive information in natural science subjects has been so frequently deplored, that it need be touched upon but briefly. For the physician this defect is gradually remedied through his further studies. Those high school graduates, however, who have no occasion in their later studies to become informed in natural science, drag their defects with them throughout their lives. This leads often enough to disagreeable consequences, which the victim does not, however, recognize as resulting from his shortcomings in this direction, but which the physician, for example, soon traces to their causes. As an illustration of this idea we may take the ignorance of the scientific facts that have led to the development and establishment of various rules of hygiene. Here it is possible to prescribe never so many detailed regulations, and to force the public to observe them: but as long as these rules are followed mechanically, without any knowledge or understanding of their significance, their utility will be in large measure illusory. But I do not wish to emphasize too much the deficiencies in scientific information found in our students: this defect may be gradually made up in the course of time. What is important is the fact so characteristic of our "humanistic" education, that the students acquire the almost ineradicable habit of resorting to books and to manuals for all information on scientific subjects, instead of looking about them at the concrete world itself. This is but the result of the year-long practice of occupying themselves with languages. This characteristic is found in its full development in the science of the scholastic period: the scholar knew and valued only what was given by Aristotle, and every problem that presented itself was solved not by an appeal to the facts of the environment, but by resorting to the words of the master. In

the same way, our high school graduates have missed almost completely the most valuable element in scientific instruction, namely, their own opinion, unless they have perchance occupied themselves with scientific matters outside of school. There results at the same time a more serious gap in the student's preparation than the deficiency in scientific information, and that is his lack in certain forms of skill. Information or knowledge can be easily accumulated when needed, but it can be just as easily forgotten. Skill, however, is not so easily acquired, but it stays longer once it is attained, especially when it is concerned with activities that are constantly exercised.

The manual indexterity of young medical students is notorious. But even this manual dexterity is but rarely the result merely of unpracticed motor innervation. In nearly all cases it is a direct result of the mental helplessness in dealing with the concrete. The student lacks in the ability to observe the object, and to exercise his judgment upon it. He is quite helpless in the presence of the simplest object, and is generally incapable of distinguishing between the essential and the incidental. It takes much time and patience to train the medical student, for whom the ability to observe is the first requisite, to observe properly and to organize his observations.

A student was asked to describe the appearance of a diphtheria membrane, and gave the reply that such a membrane was "square." To this medico the incidental form of his microscopic preparation was the essential character. This helplessness first appears when it is necessary to describe some object, or apparatus, or a plan of experiment. It might be expected that the linguistic training of the schools, which always makes use of the formal, logical mental operations, might at least impart some skill in describing simple objects; but in actual experience such is not the case.

When it comes to combining or organizing his information, the situation is much worse. Connections and relations are never recognized; the most obvious implications of the concrete facts are completely overlooked. This ability to organize the facts I consider of much greater importance to the physician than a great mass of information; and for this reason I am accustomed to present to the candidates some experimental data which they are not supposed to know, and to require them to draw conclusions. The results of such examinations are far

from satisfactory. They have large quantities of knowledge, of the kind that can be learned from books; but there is always absent a point of view, a perspective, the ability to grasp the everyday facts, the ability to analyze, to judge, to put two and two together. This ability can be acquired only after long practice in contact with the concrete.

Everywhere we run against the results of this one fact: the education of the gymnasia is dominantly scholastic, philological book-learning. Here lies the key to the shortcomings to be observed in all who have received their education in secondary schools. It may be asserted with but slight exaggeration that the very first business of the university is to undo the effects of the preparatory schooling. At any rate it must be the chief function of the university to teach what is not found in the books; that should be the best part of an academic training.

Proposals for a Modern Course of High School Instruction.

The first demand upon the high school made by modern culture is not for more education, but for better education. We must strive not so much to increase the knowledge of the pupils, as to teach them how to accomplish something with comparatively little knowledge. If our education develops the capacity of the pupils to find themselves, to grasp relations and to form judgments, knowledge will crystallize about the problem in hand as it is needed. The shortcomings in our education illustrated by my experiences with medical matriculants point out the direction in which the required capacities may be developed: *there is needed practice in dealing with the concrete.*

In this direction something may be accomplished even with the linguistic and historical material of the school. The grammatical and verbal part of the lessons may be minimized, while emphasis is placed upon the content by means of appeals to vision; this will enliven the interest incidentally since all pupils who have not been spoiled take more interest in what comes through the sense than in the abstract.[2]

But the most important step would be an expansion and a more systematic arrangement of the instruction in natural science. To this end, I consider an extension of biological instruction a prime necessity. According to the present curricula the instruction in these subjects stops too soon, and the center should

[2]The use of charts, portraits, art reproductions, coins, etc., in history and literature instruction had of course been introduced before this paper was written.

be advanced to the higher classes. Instruction in experimental physics and chemistry may be taught in the earlier years, whereas biology, insofar as it is not mere classification, presupposes a certain amount of physical and chemical knowledge. (Placing physics earlier can be made possible only if it is treated as an experimental science, and not as a branch of mathematics.)

In regard to physical and chemical instruction there is one point that I wish to emphasize further, and that is the great importance of increasing the use of experiments. It is only in this way that the defects in the present high school instruction can be remedied. The most desirable condition would be one that allows the pupils to carry out their own experiments under suitable supervision. The value of experiments performed by the pupil himself coming in direct touch with his materials, is incomparably higher than that of the neatest and clearest demonstration of an experiment. The simplest experiment, a simple reaction in the test tube has a great educative value. It is only with the experiment carried out by himself that the pupil learns to observe and to reason carefully, logically and critically.

The greatest need, however, is in the improvement of biological instruction. Biological sciences deal with the most complex objects, and they are therefore best adapted to develop the higher mental processes, skill in recognizing and grasping complex relationships, readiness in perceiving and organizing connections and relations. The center of biological instruction must for these reasons be in the middle and upper grades; at present it ceases just at the point where its most important task should begin.

The plan commends itself of confining the biological instruction in the earlier years to exercises in observation, in which the pupil becomes familiar with the more important forms, the gross anatomy, the habits and the economic relations of plants and animals, and also the geographical distribution of the more useful organisms. The end of the instruction is here chiefly practice in observation, and there should be required original descriptions and drawings of what has been seen and observed.

In later years I would require an introduction to comparative anatomy, including man, to the history of the development of the organic world; and to paleontology. Here the pupils should become acquainted with the doctrine of descent as well as with the mass of facts upon which the doctrine of descent in the

organic world rests. The study should include an outline of comparative morphology in the modern sense, a knowledge of the cell in plants and animals, and a rapid survey of the evolution of different plant and animal types, studied in the natural order and including important fossil forms and man. Care should of course be taken to avoid the danger of treating types in too great detail; there must be no fruitless accumulation of disconnected facts. As the aim of instruction in this stage I would regard (apart from getting the idea of relation among the different forms of organisms) the practice in making comparative observations, as well as the habituation of the pupil to the concept of *becoming* and change in concrete objects. It is an unusually important task of this stage, with respect to the method of thinking, to cultivate in the pupil the habit of seeking to understand a phenomenon by finding its simplest expression, through comparison, in order to recognize what is fundamental, and to distinguish the essential from the incidental. I know of nothing in the literary curriculum that can remotely approach in value this special educative aspect of biological instruction.

For the highest classes there is required instruction in physiology. This division of biology deals with the most complex relations and presupposes a certain knowledge of physics, chemistry and morphology. It can therefore be treated only in the highest grades. I consider the treatment of physiological foundations so important in relation especially to the development of certain concepts and methods of thinking, that I would require it unconditionally of all high schools. First of all there should be developed the general ideas of the transformation of matter and of energy, of the conditions essential to life, stimulus and reflex, and the interdependence and coöperation and coördination of the different parts of the differentiated organism. This study should also be comparative and deal with the structures and processes in simpler plants and animals first and lead up to human physiology. In the study of human physiology hygienic application should of course be pointed out; at the same time it is necessary to guard against the danger of giving the pupils the idea that they are able to dispense with medical advice in cases of sickness, or that they are even prepared to do their own doctoring. This danger is to be avoided by the method of instruction; the pupils are to learn that in getting the rules of hygiene they are not getting medical knowledge. The

aim of instruction at this period is the development of the ability to grasp and to analyze more complicated and constantly changing relations, and to understand the interdependence of single processes in a large complex, as well as the effects of each upon the total in a changing whole. Practice in these mental processes has an extraordinary educative and practical value, for everywhere the individual meets in his daily life, in his political and in his industrial relations, changing systems, that he must comprehend properly in order to be able to deal with them efficiently.

The individual cannot get sufficient practice in dealing with such complex relations. Adequate material for instruction in the requisite mental processes is furnished by the proper study of the conception of metabolism, its dependence upon outward conditions, its modification by stimuli—in short, occupation with the processes of the living organism. The pupil should become accustomed, through such instruction, to contemplate the apparently stable organism as constantly changing, and to think of suitable ways of modifying the direction of such changes in a complex system. These activities are of a kind that he will need in his later life; these points of view will enable him to grasp the complex situations with which he will later have to deal.

However, the full value of these studies can be secured only through direct dealing with the objects studied. In all grades therefore, there must be insistence upon demonstrations and experiments. It would be desirable if special laboratory hours could be arranged, during which the pupils are to come as much as possible in immediate touch with the materials studied. In all grades also arrangements should be made for scientific excursions, the time for which should be credited to the teachers the same as work in the class room. These excursions are better adapted than regular class room exercises to teach certain things and to develop certain habits, because of their freedom from restraint and because they bring the pupils into close touch with the world outside. These excursions should not be confined to the observation of some particular class of objects, but should serve rather as means for bringing together all that is experienced into some coherence and relationship.

In all science teaching the chief aim should be the cultivation of the pupil's own view, through occupation with the concrete.

SOME FALLACIES OF BOTANY TEACHERS.

By Joseph Y. Bergen.

A little observation and reflection would probably afford material for the discussion of unwise choices of subject matter and unsound methods in teaching any one of the biological or the physical sciences. The writer here speaks of secondary school work in botany merely because he has thought more about it than about any other science subject. Most of the views here criticised have eminent advocates and it would not be possible in a brief paper fully to demonstrate their mischievous influence. But it may be worth while to try to suggest the errors in a few of the most important of them. Very often the mistaken policy adopted is not the fault of the teacher but of his official superiors.

Uniting the three subjects of botany, zoölogy and human physiology under the common title of biology is one of the most plausible suggestions for time saving and simplifying in that section of the high school work. Everybody knows that cytologists are coming to feel the likenesses more and the differences less in studying animal and plant cells. But our high school pupils are only to a very limited extent cytologists. We must recognize the same fundamental differences between the structure and functions of the higher animals and the higher plants that Linnæus did. And just as the earlier naturalists were primarily interested in the higher organisms, every boy and girl is likely to be so interested because he or she is a beginner, as those pioneers were. In the study of the human animal the emphasis is so different from that in botany or ordinary zoölogy that linking human physiology and hygiene to them helps but little. If the botany teaching were to be shaped so as to make the most useful foundation for the subsequent study of human hygiene and sanitation, a wholly disproportionate amount of time would have to be spent on such topics as the chemical composition of the cereals and of leguminous seeds, the life habits of bacteria, their sensitiveness to toxic agents and means of destroying low organisms in drinking water. The work in botany would be made one-sided and unsatisfactory for the sake of putting the pupil in a condition to get the most possible out of a subject of decidedly less educational value later on in his course. And a similar difficulty inheres in making the zoölogy

tributary to the study of man as an animal, with a nervous system to watch over and a digestive apparatus to cherish.

There is beginning to be a vigorous demand, perhaps most noticeable in parts of the middle west, for a highly "practical," i. e., economic, kind of instruction in botany and zoölogy. It is felt that, for one thing, the teaching should be so shaped as to make use of the commonest garden and field plants to illustrate plant anatomy and physiology. Of course no teacher in his senses would hunt up a rare greenhouse orchid to demonstrate a point which could be equally well shown by the use of a garden lily, a hyacinth, or an onion. But, as one of the ablest botanists whom I know pointed out to me in conversation not long ago, there is a very specious fallacy in the unqualified insistence on the use of common material. The pupil is to study, let us say, stomata. The cabbage is a most familiar plant, therefore let us make stomata easy for him by giving him cabbage leaves to histologize. Now a single trial would convince any unbiased teacher that the familiar cabbage leaf is not nearly as easy a subject for the study of stomata as are easily peeled leaves like those of the iris or firm ones for cross sectioning, like those of *Cycas*. So, too, the fact that the common bean is a highly useful plant and *Sedum* or *Trillium* is not would still leave the bean flower much the poorest of the three with which to begin the study of floral structures.

A still more radical phase of the movement toward economic biology appears in the demand for lessons on all sorts of topics bearing on horticulture and farming, from injurious insects to plant breeding. Doubtless in some country high schools a good deal of such work can be made thoroughly interesting and profitable. And in any schools such matter, in very moderate amounts, may properly be assigned for supplementary reading. Leaving out business and other technical courses, however, when one begins to make economic considerations the measure of educational values he begins to pile up absurdities. As soon as the teachers of geography, history and geometry are willing to bend most of their respective efforts toward instruction regarding commercial routes, the alternation of periods of activity and depression in the world's business, and mensuration, it will be time for biology teachers to consider favorably corresponding pseudo-utilitarian innovations. But if the most valuable crop that any country can produce is intelligent men it must follow

that any kind of study which is preëminently suited to cultivate habits of careful observation and orderly thinking in school children is especially important. Then that kind of biology which gives young people some adequate conception—partly obtained from their own field and laboratory studies—of the animal and plant inhabitants of the earth, is better worth while than that which primarily leads to more abundant hay, grain, butter and pork making. In other words, we can develop the faculties of a boy faster and further (and therefore do more for the world) by setting him to work on the structure and functions of the corn plant than by making him count and weigh the kernels of a half dozen ears of as many improved varieties of corn. Such counting and weighing, unless they form part of an extended, systematic investigation carried on by the student, have no more educational value than keeping tally of the loads of coal sent out by a fuel company.

There has been among teachers of botany an idea, now fast vanishing, that ecology is at once the easiest and the most interesting department of the science. It is hardly necessary to explain to anyone who has even dipped into such books as Warming's *Œcology of Plants,* Weisner's *Lichtgenuss der Pflanzen,* or Clements' *Research Methods in Ecology,* that thorough ecological studies demand of the student considerable knowledge of subjects ranging all the way from taxonomy to physics. High school pupils can learn a few useful facts about such matters as heliotropic and geotropic movements of plants, the occurrence and meaning of deciduousness among trees, insect pollination, competition, the concept of a plant formation and a plant association. Further they cannot profitably go.

Though the belief that plant ecology is "easy" is obsolescent, an equally pernicious notion that plant physiology is "hard" still prevails. It has, in some instances, gone so far as to lead to something perilously near to the complete omission of the subject from the text-books and the class work. Of course the more recondite matters, such as the causes of the movements of liquids in the plant body, the precise function and *modus operandi* of stomatal movements, the details of sexual reproduction in many groups, and a host of other topics are difficult enough to tax the energies of a Pfeffer, a Strasburger, or a De Bary. But there are so many simple, manageable things for the young beginner to work out! It is far easier for him to

discover for himself the fact and roughly to measure the amount of transpiration, to prove the dependence of starch production on light, and roughly to ascertain the temperature limits within which germination of a given kind of seed is possible than to learn by his own observations anything worth while about fibro-vascular bundles or even to master the details of pollination in *Asclepias* or most orchids.

A few words should here be said about the very prevalent idea that, since plants have been evolved from the unicellular condition to that of the most complicated assemblage of structures found among seed plants, the pupil's knowledge of them should be gained along the same road. Perhaps with students of twenty this might be true, though one of the best all-round teaching professors of botany whom I have ever known, found that his classes of college beginners in the subject could not do anything like the year's work when they began with the cell as a unit that they could and did when they began with readily visible and somewhat familiar forms. It is doubtful whether the English-speaking world has ever known a more successful teaching biologist than Huxley and there are still some of us who remember how he reversed the order of treatment in his *Biology,* after a thorough trial of the evolutionary order in the first edition. The fact is that (as any teacher can find out for himself, if he cares to experiment with his classes on the matter) pupils of high school age do not at once or even very quickly grasp the meaning of things seen only by aid of the compound microscope.

Somewhat akin to the error of feeling obliged to follow the history of plant life with the learner from his very beginnings of botany, simply because it was the evolutionary path, is the blunder of insisting throughout on scientific terminology and of supposing that each new term learned is a new idea mastered. It is easy to say that the golden mean is to use just as many technical terms as are necessary for accuracy, and no more. But who shall decide precisely where the line is to be drawn? To me it has always seemed a wrong done to the learner to give him a specially coined Greek derivative where a single English word or a manageable compound will serve. Seed-plant, rootstock, sac-fruit, for those who are not and are not to become technical botanists, are just as good terms as spermatophyte, rhizome, and ascocarp, while they are far easier to learn

and to remember. It is indeed a pity that we have not a host of simple terms like the German *Keimblatt, Markstrahl,* and so on, but let us use what we have.

One more current delusion and the list is completed—not of all those to which teachers of botany are subject but of those to be mentioned in the present paper. This is the most pernicious notion that anyone can teach botany. It is only now and then that an unfortunate man or woman who never has studied the subject at all is called upon to teach it, though one cause of the sort was not long ago brought to the writer's notice. But it is common enough to assign teachers who know less botany than is contained in Dr. Gray's charming little *How Plants Grow* to the charge of botany classes in a moderate sized high school. A delightful example of the results of this plan was the case of the teacher in a large high school who had started to determine *Tropæolum* with his division by what his colleagues satirically called the indexical method. Turning to *Nasturtium,* at the page given in the index, he found to his dismay that it was an aquatic or marsh plant, with small yellow or white flowers. This was long before the formulation of the Vienna code. Something had to be done, so, with a hasty excuse to the class, he rushed out of his own room into that of another teacher and there learned that what laymen call nasturtium is known to botanists as *Tropæolum.* But, seriously, it is no easy matter to teach high school botany well. Unless the instructor knows a good deal about plants, what they are, how they are built, what they do, and how and (partly) why they do it, and knows also fairly well what his pupils are seeing and what they are thinking about the work in hand, he will accomplish little. Most of us have known dozens of botany teachers, but we could count the supremely successful ones—in school or college—on the fingers of one hand. If the time should ever come when most secondary schools are willing to devote at least a year to botany, to give all reasonable facilities to teachers of the subject and in turn to demand of them as adequate preparation as is required of a teacher of Latin or geometry in a first-rate fitting school, we would surely find that the educational value of botany is greater than most of us have ever ventured to rate it.

BIOLOGY IN A PENNSYLVANIA HIGH SCHOOL.[1]

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We are beginning to see that it is the truths of the *living* world that are destined to control us; and that great as is our interest in the harnessing of the air forces or in the various possibilities of radio-activity, this interest is slight compared with our longing for an anti-toxin in tuberculosis and typhoid fever. *Life* is the keynote of the coming science and she will not be ignored.

We have gone rapidly in our teaching from the theoretical to the practical. In the old days, our boys and girls committed to memory pages and pages of ideas based on other people's investigations. Their absolute ignorance in the matter of falling bodies or expansion of gases has produced the laboratory method. And how did we formerly teach the children to care for their bodies? We said, "Children, if you do not bathe often and well, you will not be strong." Now we actually put the child into the tub, provided his mother is not inclined. We used to say, "John, you ought to go to the doctor and have your throat examined." Now we say, or hope to say, "Doctor, step this way and examine John's throat."

But what about biology? What is it? In its broad sense biology includes all studies relating to living matter, every phase of plant study and of animal study being included. Man himself is the climax, the study of the human mind constituting the pinnacle.

Have these studies of plants and animals, and most of all of man himself, any place in the general education of the average citizen, or should they be delegated to the few enthusiasts and specialists? Is it not true that in early life the world of nature is the child's world? His early education is almost entirely nature study, seeing, and hearing, and feeling, and then doing. Is there any good reason why he should cease to be interested in his immediate surroundings just because he has entered school? Why stop nature study when the child begins school? He must stop noticing things and spend his time trying to express thoughts which he never had. They call this *reading*. Why should he not have some thoughts of his own, about things that

[1]Paper presented at the Cambridge Springs meeting of the Northwestern Pennsylvania Science and Mathematics Teachers' Association.

interest him, and then learn to express these thoughts? The burden of proof is entirely upon those who would stop nature study when the child enters school. Nature study develops observation, thought, and expression. What more can we ask of any subject? But best of all, it develops sympathy with every living thing, love for all creation. It gives genuine enthusiasm toward life. Are there any people more happy or contented than those who feel friendship for the plants and insects, birds and brooks, and hope in the stars above? And is it not true that our boys and girls who delight to roam around the woods to "see things," as they say, are the boys and girls who scarcely ever go wrong? They remain wholesome, for they belong to the universal brotherhood.

At the present time many schools in this state do not attempt to teach anything regarding the plants and animals that are living right alongside of us. There are some schools where a child never sees, or hears mentioned, a single live thing, unless a chance supplementary reading introduces a slight discussion. The boys and girls leave the elementary schools, pass on through the high school, and still their minds are kept tightly closed against all living intruders. And worse still—the state law notwithstanding—they do not hear mentioned anything in the way of hygiene, because when they reach the high school they are to have a half year's "cram" in somebody's physiology. Ridiculous, and even outrageous, this seems to one giving the matter any thought, for so many boys and girls never step inside the high school. And suppose they do, what is a paltry half year in learning of things so closely affecting their lives? Will the time ever come in this state when our grade schools will teach the little folks hygiene and nature study, and our high schools will teach real biology, and real physiology and hygiene, to each pupil—that is, not making it elective? And should not a high school be able to maintain a year in zoölogy and a year in botany which shall be equivalent in educational value to a year in chemistry or in physics?

We are now trying to introduce into the first year of our high school a course in biology which is essentially nature study. Of necessity it must be, since there is no foundation for real biology. When this work shall have been absorbed by the grades, we shall change to the science of biology. In this course we are not attempting to teach the development from simple to higher forms.

We shall begin with insects. The field is boundless and the early fall gives great opportunity for enthusiasm. The aim must be to teach what everyone ought to know about a few of the most important insects. We shall study them as insects of the household, of the garden, of the field, and of the forest. We must provide nets and all the appliances for collecting. We must make excursions and try to get life histories developing and made into a collection. We shall have definite drawing and written expression in the notebook. We shall introduce into the room an insect chart, to be filled in after the manner of a calendar; that is, the name of the insect, the name of the child, the date, the order of insects, and the type of metamorphosis. We shall have four charts to be filled out by the pupils during the year: the insect chart, a bird-calendar, a flower calendar, and a tree calendar.

After the study of insects will come the flower. Probably by this time we shall be obliged to obtain material from the greenhouse in order to get a simple flower, since the wild flowers become more complex as the season advances. This study of the flower gives opportunity for teaching life processes which must enter into all the remaining work of the year. Our educational bee-hive will help in the study of the relations of flowers and insects.

Then will come the study of trees, in the city and in the woods. We shall plant tree seeds at school and at home in order to encourage the *planting habit*. We shall make collections of seeds and dry fruits, giving attention to seed dispersal. We shall fill out our tree calendar which will bear record of the time of fruiting, and again in the spring of the time of flowering. Pupils who care for it will be encouraged to make a collection of leaves of the trees of this locality. Possibly some pupils will be interested in labeling the trees of our city parks. The economic aspects of the subject must not be ignored, and we shall have simple experiments relating to forestry.

We shall now enter an entirely different field. The study of the amœba and other one-celled animals and some one-celled plants will open up an entirely new world. We possess enough microscopes and can make this worth while. The processes of nutrition, sensitiveness, movement, and reproduction become comparatively easy in this connection. We are then readily introduced to the yeast plant and to bacteria. After seeing the bacteria of a hay infusion it is not difficult to get into the

subject of diseases. Here comes in the instruction regarding tuberculosis and typhoid fever, after which comes the yeast plant and alcohol, where we like to put considerable stress. This work is mainly experimental, while the later work with the frog gives opportunity to complete it.

We shall now introduce birds and try to develop enthusiasm for birds in winter. This is certainly the most exhilarating teaching of the year. Mr. Chapman's method of teaching them by the four groups is most practical: the permanent, the winter, and the summer residents, and the transient visitants. Before the first robin or song sparrow appears we shall show stereopticon views, which may be obtained from Harrisburg free of cost. The *Audubon Pamphlets* and *Bird Lore Magazine* will be used. We shall have taken in a little time in the preparation of a bird notebook by each pupil, containing lists of the birds which they may hope to see during the year. This will have been necessary in order to begin our bird calendar in the fall. We shall make bird houses, putting them up at home and at school. They will surely get wrens and possibly martins. And we shall have material for an Arbor and Bird Day program.

By this time the frogs and wild flowers will be awake and we must give attention to buds and germination. We shall take the flowers first, and then the seeds, and then the opening buds. Excursions will furnish material for our flower calendar which was started in the fall. There must be laboratory study of several types, with attention to plant families and societies and to the actual growing of a plant by each pupil. Germination of two or three types of seed, experiments in germination and growth, with some attention to the opening buds and their structure, will occupy us until the frog demands our notice. There is no animal so well adapted to laboratory study. By this time, and in fact some little time before this, we shall have gotten out of nature study into the science of biology. We shall study the frog with special reference to the human body, paying special attention to the nervous system, and to matters of hygiene.

We are perfectly aware that the course outlined above does not agree with that of any published text-book, but it seems to fit our needs and we shall attempt it. If the time shall ever arrive when there shall be complete correlation between grades and high school in the matter of the sciences, we hope to see something like this: nature study from the first year through the

grades; hygiene and physiology introduced at the fourth year, as in Dr. Gulick's admirable series, and continuing through the grades; elementary science in the first year of the high school, consisting of the rudiments of botany and zoölogy in the first half, and the rudiments of chemistry and physics in the second half, the work up to this point being compulsory for all pupils; then finally the necessity of choosing at least one more year of science in the high school, physics, chemistry, botany, zoölogy, or human physiology. In addition, there should be lectures in hygiene for boys and for girls, and for teachers, given at intervals of the high school course, with complete medical supervision throughout the whole period of school life.

It may seem that we are trying to displace the Three R's in the curriculum, and to give science more than her share of the time. However, it is a fact that as matters stand at present we fail to teach many things which our boys and girls ought to know, mainly along biological lines. Failure to teach some of these things is responsible for untold misery. And misery is mostly biological, as we have noticed.

Apart from this, we believe that we ought to give our boys and girls the foundation of happiness. The nature lover is happy indeed. If he have also the ability to keep himself well and strong, his happiness is insured. To him the seasons come around like old friends; the birds sing to him; as he walks, the flowers stretch out their faces; and as each year fades away, he looks back to a store of happy memories.

In the October *Arboriculture*, published by John P. Brown at Connersville, Ind., there are some unusually fine photographs representative of the use of the hardy catalpa (*Catalpa speciosa*) in tree plantations in different parts of the country. Mr. Brown is one of the leading enthusiasts on the hardy catalpa, and by means of his paper has brought many people to prove the value of the tree for practical planting. For eight years Mr. Brown has labored to bring before the people the needs of forestry and by his constant urging the virtues of the hardy catalpa, has been a genuine public benefactor. It is with much regret that we read in this issue of the *Arboriculture* that the author will discontinue its publication at this time. Mr. Brown's future address will be Bay Minette, Ala., where he will live on his forest experiment farm, Arboretum. We wish to express the hope that Mr. Brown will continue to keep before the public the merits of the tree to which he has given so much attention and which owes much of its public reputation to this one man. O. W. C.

REPORT OF THE COMMITTEE ON SECONDARY SCHOOL GEOGRAPHY.*

By James F. Chamberlain,
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The expression "the new geography" is now so infrequently heard, that it may be of value to remind ourselves of its former significance. The appointment by the National Education Association in 1892 of the Committee of Ten, was an important event in the development of school subjects in general, and of geography in particular.

The report of the Conference on Secondary School Geography crystallized and gave definite and authoritative expression to the belief, which for a number of years previous to this had been a conviction on the part of the few leaders in the subject, that earth science should be a broader, deeper, more helpful study than it then was. It regarded the subject not as a mere description of the most striking features of man's physical environment, but as a study of the evolution of environment and its relations to human conditions. It recognized the fact that if geography was to maintain a place in the secondary school as a subject for serious study, it must experience a decided change in character. As the laboratory method was being used with success in chemistry, physics, and the biological sciences, it was considered wise to apply it to geography.

The close relations between geology and geography, the fact that the former was quite well organized and that there was material available for laboratory work, together with the personnel of the conference, all combined to give the movement a decided trend in the direction of geology or physiography. Because there were no text-books that treated the subject from the new point of view, the Conference recommended that physical geography, along the lines then followed, be presented in the first year of the high school, and physiography or geology in the last year.

In 1895 there appeared the first modern American physical geography. It presented geographic forms and processes very fully, but practically speaking, with the exception of a brief chapter used in the nature of a summary and entitled, "Man and Nature," it ignored their influences upon human life.

*Given before the Science Department at the Denver Convention, July 9, 1909, of the National Education Association.

A comparison of this book with its predecessors, as well as with the more recent texts, is very interesting. The earlier books devoted considerable space to a description of the surface features of the several continents. Comparatively little attention is given to the origin and classification of land forms, while there is practically no reference to human response to its environment. Their treatment of the atmosphere is much briefer and less valuable than it is in the text published in 1895.

In 1897 the Science Section of the National Education Association appointed a committee of nine to consider physical geography in secondary schools. The report of this committee, made the following year, was in general along the lines laid down by the Geography Conference. It is interesting to note that of the five men who signed the report, three were professors of geology. The report states that "the subject should be carefully held to the leading idea of the physical environment of men." The committee favored elective courses in geology and meteorology.

There has been marked advance in the subject of physical geography during the past sixteen years. We have now more than one half dozen good texts, and as many laboratory manuals. Our equipment has been improved and teachers are receiving special training along this line in our normal schools, colleges and universities.

Although the economic and commercial phases of geography are receiving an increasing amount of attention, secondary school geography is to-day practically physiography. Therefore, unless otherwise specified, all references here made to geography as now taught in the high schools, have to do with this division of the subject. The following are the more important reasons for considering a change in the nature of the course imperative:

1. The course, as at present organized, places too much emphasis upon the detailed study and classification of land forms, and too little upon human response to these forms. The amount of space devoted to the lands by the various physical geographies varies from 40 per cent to 76 per cent of the total. The criticism here made, however, is not that too much space is devoted to the lands, as one of the four great divisions of physical geography, but that the lands are not sufficiently studied from the geographic point of view, namely, the human.

2. A concrete study of human response to its environment does not receive sufficient attention. This point has reference to all of the divisions of the subject. Your committee recognize the fact that a comprehensive study of this subject, as applied to large areas, is impossible in a brief course devoted to physical geography. The omission of such study from secondary school geography is a serious matter, however. A careful examination of our texts will show that the facts and principles stated are given scant application to concrete areas. The newer books show an improvement over the old ones in this respect.

3. The course aims to fit the student for college rather than for the affairs of life. Secondary school geography, as a part of our educational scheme, should agree with the latter in so far as general function is concerned. In other words, it should aim to render the greatest possible service in preparing the student to meet successfully the opportunities and obligations of life. The fulfilling of college entrance requirements—a matter which now receives serious attention—should receive practically no consideration. The soundness of this statement becomes evident when it is remembered that only about ten per cent of the total enrollment in public and private high schools graduate. In other words, practically *all* graduates of secondary schools enter at once upon some business, some profession, or the duties of home life. It is to these students, more than 90 per cent of the total number, that the secondary school should devote itself heart and soul. The popular but false conception that the high school should place great emphasis upon meeting college entrance requirements in geography as well as in other subjects, is responsible for certain conditions in the curricula of our secondary schools that are much to be regretted.

4. Secondary school geography does not give the student a grasp of the natural resources, the industries and the commerce of the world. This condition is, of course, inevitable so long as the geography in the secondary school is almost exclusively physical geography. The training which this phase of the subject, when properly presented, affords is too valuable to be neglected. Especially is this true as applied to our own country.

5. Geography in the secondary school does very little as a preparation for the teaching of that phase of the subject which, in the elementary school, receives chief attention. Your committee recognize the fact that it is not the special function of

the high school to prepare its students for any particular occupation or profession. But as the training of children is the most important work in which anyone can engage, most important both from the standpoint of the home and the nation; and as a considerable number of teachers is yearly drawn from the ranks of high school graduates, we conceive it to be a part of the function of the high school, until such time as professional training for the work of teaching is universally required, to give its graduates such work as will best fit them for teaching, provided that it is at the same time of great value to the many who will not teach.

We do not imply that a good course in physical geography does nothing toward preparing the graduate of the secondary school to teach in the grades, but it omits those phases of the work essential in the elementary school, and which are at the same time of most importance to the average member of society.

6. Secondary school geography, as at present constituted, cannot give the student that knowledge of the regions and peoples of the world which intelligent participation in the affairs of life requires. If broadly interpreted this encompasses all of its weaknesses. By its very nature high school geography places a handicap upon the student who tries to do advanced work in the subject. The lack of a knowledge of place relations, and of regional geography in a broad sense, is a weakness so universally shown by students in the entering classes of both normal schools and colleges, as almost to discourage teachers of geography in those institutions. It is this knowledge which the average person, no matter in what walk of life, most needs. The farmer, the fruit grower, the stockman the miner, the promoter of railroads or waterways, the manufacturer, the shipper, the fisherman, the artisan, the health seeker and the traveler, all feel its need daily. We cannot read with intelligence the daily papers and the periodicals, not to mention history and literature, without some considerable knowledge of regional geography.

We present the following as essentials of a course in geography for secondary schools:

1. Those parts of mathematical geography which show most clearly how human life is influenced by the relations between the earth and other members of the solar system. Such points as the arrangement of the mathematical and heat zones and the varying boundaries of the latter; the change of seasons; latitude

and longitude; standard time and the International Date Line are important. Facts as to the size of the members of the solar system, their distances from the earth or sun, and their periods of rotation and revolution are not considered essentials.

2. First in importance among the factors influencing life is climate. Therefore, atmospheric phenomena should receive careful attention. The principles should be applied as fully as time and the ability of the students permit. The topics which should receive chief attention are the following: (a) The conditions determining the temperature, pressure and humidity of the atmosphere. (b) The great atmospheric movements. (c) Storms, especially temperate latitude cyclones, studied by means of the weather maps, and their relations to crops, floods and transportation. (d) Precipitation; its causes and distribution; and its influence upon occupations and habits of life generally. (e) Weather changes, such as the effect of unseasonable frosts upon crops, and the efforts of man to prevent the damage. The influence of blizzards upon stock on the western ranges, and telegraph and railroad business in many parts of the country. The effects of storms upon wheat, oats, hay and other crops. (f) We urge the importance of a study of the work of the Weather Bureau, having students present specific illustrations of the value of its work. Comparatively few realize the multitude of human interests that are advanced through the operations of this bureau.

3. A brief study of the ocean as a modifier of climate, as an agent in the destruction and construction of land forms, as the source of certain commodities, and as a medium for the transmission of the commerce of the world. A detailed study of ocean depths, of temperatures at various levels, of tides, of the character and distribution of ocean life, may well be omitted.

While our texts treat the ocean as a separate division of physical geography, we favor an incidental treatment, with a brief summary. The influence of the ocean upon climate—the topic of chief importance—should be considered in connection with the study of climate. Erosion and sedimentation along shore lines should be treated under those topics. The commodities obtained from the ocean, as well as the commerce which it bears, would naturally receive attention as a part of commercial geography.

4. The larger geographic forms such as plains, plateaus,

mountains, valleys, rivers, falls, lakes and glaciers should receive careful study. Human interests and activities are largely confined to the lands, but it is obviously of far greater importance that we should understand our relations to geographic forms than that we should have a thorough knowledge of their evolution, or be able accurately to classify them.

Graduates of secondary schools should know the location of the great plains of the world, and how they are related to the production of food, of occupations, of transportation and the distribution of population. Whether or not these students can name the six or eight classes of plains given in our text-books is a matter of very little importance. Students should understand how certain definite mountains influence climate, the distribution of plant life, human occupation of their areas, the construction of railroads, the use of streams for transportation and the development of water and electric power, how they served as national boundaries and have helped to mould national characteristics. The ability to name the different types of mountains, and to classify faults and folds, is of little value to the average person, however. In other words, it is the human point of view that is important as applied to every topic.

Students should be encouraged to discover human response to its environment in the home area, as this gives reality to the subject and prepares them to work out and appreciate these relations in remote areas.

The amount of emphasis placed upon the study of the various geographic forms and processes will depend, in part, upon the location of the individual school. A school situated in a mountainous region would very properly devote more time to the consideration of the influence of mountains upon life than would one in the prairie section. In the first named area the relation which mountains bear to climate, industries, settlement, and road building, are relatively of greater importance than in the second, because they are at hand and are therefore more meaningful.

5. A study of the larger features of the resources of our country, such as its soils, waterways, water powers, forests and mineral wealth. These features of our geographic environment are so vitally connected with the daily life of every individual as well as with our national progress, that ignorance of them is a serious matter.

This study should show the distribution of the resources, their accessibility, their relation to road building, to distribution of population, to development of industries, to location and growth of cities, to commerce, and to social conditions. The work of our government in modifying geographic environment should receive careful consideration. Our government is expending vast sums of money in carrying on soil surveys, in improving plants and animals, in reclaiming desert and swamp lands, in the preservation and extension of forests, in developing waterways and harbors. These are subjects upon which every man and woman should be informed from the geographic point of view. The value of such work in the moulding of useful members of society is certainly very great.

6. A knowledge of the general geography of the most important countries and peoples of the world.

The grasp of regional geography obtained in the elementary school is necessarily very meager. Geography is quite generally discontinued in the seventh grade, and, as has been stated, practically no attention is given to regional geography in the high school. A knowledge of the geography of our own country and of Europe is a much-to-be-desired factor in good citizenship. A somewhat detailed study of these two regions would incidentally put the student in possession of considerable knowledge of the other continents. In addition to this he would gain a "geographic consciousness" that would be of great value.

If it be true that a large part of the education of the average individual comes through reading, then it is of the utmost importance that we should be able to read intelligently. This, in the fullest sense, is impossible unless one's knowledge of geography is wider than that offered by the present high school course in this subject.

7. Some conception of how the history of nations has been shaped by geographic conditions.

We do not forget that this should be a vital part of all courses in history, but it should also receive very definite attention in secondary school geography. All nations afford illustrations of this, some more than others. That our own country is a fruitful field is evidenced by such works as Brigham's "Geographic Influences in American History," and Miss Semple's "American History and Its Geographic Conditions."

8. The ability to trace, in the large, the relationships between the most important geographic forms and geographic processes.

and to appreciate the responses which human life everywhere makes to its physical surroundings.

Only as the student has observed the results of geographic processes on a small scale can he have any definite conception of the evolution of larger and distant geographic forms. When the student appreciates the significance of human response in the vicinity of his home, he has laid the foundations for the discovery and interpretation of this response in remote areas. Moreover, this training will furnish some basis for seeing in advance the general trend of the geographic development of a new region.

In order to make it possible to present the essentials as herein outlined, your committee make the following recommendations:

1. Geography, touching as it does the daily life of every individual at so many points, should be, in some form, a required subject in all secondary schools.

Sixteen years ago the Committee of Ten recommended that physical geography be required in all high school courses. While we recognize the importance of physical geography, we feel that this is too restricted a view, and that, as already pointed out, it does not include those phases of geography which are of greatest value to the average individual.

2. The subject should be pursued for not less than one year.

While more than one year is very desirable, we feel that through careful selection of topics and the elimination of unimportant details, very much can be accomplished in the time stated.

3. The subject should be presented during the first year of the high school course.

Such an arrangement will connect the work with that of the elementary school with the least possible break in time. It will also help the student in such work as he may take in history, literature, economics, geology and meteorology. A third advantage is that it will reach a larger number of students than it will if presented later in the course.

4. There should be at least five recitation periods per week.

5. About one fourth of the total time should be devoted to laboratory and field work. This should by no means be confined to the study of physical geography. Much laboratory work and some excursions should be undertaken in connection with the commercial phase of the subject, and there should be carefully planned exercises based upon maps and models.

6. We recommend that about one half year be devoted to

the larger topics in physical geography, with the human side made more prominent than at present, and that the remainder of the year be given to a study of North America and Europe.

Such a course would give our students some considerable knowledge of the geography of the most important countries of the world, and incidentally much useful information, as applied to other areas, would be acquired.

The chief difficulties that stand in the way of putting such a course into operation are two: first, time-honored custom, and second, the lack of text-books that meet the requirements. We believe that careful consideration of the subject from all points of view will remove the first, while the publishers will provide us with the necessary texts almost as soon as we declare ourselves ready for them. In the meantime, the first part of the course can be carried on by means of selected portions of our physical and commercial geographies, numerous government publications, and periodical literature. In the study of North America, the International, Russel's North America, The Lakes of North America, The Rivers of North America, The Glaciers of North America, and The Volcanoes of North America; Brigham's Geographic Influences in American History, Miss Semple's American History and its Geographic Conditions, and numerous government publications are available. Helpful material for the study of Europe is to be found in the International, Britain and the British Seas by Mackinder, Central Europe by Partsch, Hettner's Länderkunde, Volume 1, Europe, and government publications.

The work of your committee has been carried on under great difficulties. Its members live in widely separated parts of our country and hence there has been no opportunity for conference. While we are not unanimously agreed upon every point herein mentioned, we find ourselves in hearty agreement on the report as a whole. If the conclusions here reached are pointed in the right direction, and shall help to bring about a more fruitful study of one of the most practical as well as one of the most broadening and humanizing of subjects, we shall have been well repaid.

James F. Chamberlain, Los Angeles, *Chairman*.
W. L. W. Field, Milton, Mass.
Mrs. Martha Krug Genthe, Hartford, Conn.
Mark Jefferson, Ypsilanti, Mich.
W. L. Moore, South Hadley, Mass.
W. H. Snyder, Hollywood, Cal.
R. H. Whitbeck, Trenton, N. J.

AN IDEAL COURSE IN BIOLOGY FOR THE HIGH SCHOOL.

By Herbert E. Walter, Ph.D.,
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(*Continued from November.*)

Under Topic 4, The Relation of Plants to Animals, there may be discussed, first: helpful relations as in the case of the transfer of pollen by insects; the distribution of seeds by animals, and instances of symbiosis; second: neutral relations, by means of which plants and animals may be associated together without of necessity being either a benefit or an injury to each other, and finally, hostile relations. Under this latter head a consideration of the injury done to plants by animals is suggested and the adaptations of plants against such injuries. Autumn is an especially favorable time for observing the relation of insects to vegetation—particularly to forest leaves. The rather unusual cases of carnivorous plants would naturally be taken up under this topic and particularly that very important phase of plant parasitism upon animals exemplified by the pathogenic bacteria. Perhaps this would be the place to develop the essentials of bacteriology, which should surely be included in every "ideal course in high school biology." The fundamentals of bacteriology lend themselves easily to laboratory exposition even without the technical paraphernalia of a bacteriological laboratory. The immense practical importance of correct ideas about man's relations to his microscopic friends and foes, around which so many brilliant biological discoveries have centered and are bound to center still more in the future, demands a place in the training of every well-informed citizen.

Topic 5, Animals and Plants in their Physico-Chemical Relations, is a topic much like topic 1 except that it deals with animals more particularly than with plants. Animal behavior, that is, the adaptations of animals to the physical factors in their environment such as temperature, light, gravity, etc., may be laboratory-ized to a considerable extent especially with the lower forms, which react more directly to the changes in their surroundings than do the higher forms, the inhibiting action of whose more elaborate nervous system is likely to complicate results. The movement of copepods or flatworms with reference to light; of slugs on a sheet of glass or banana-flies in a glass tube with reference to gravity; of fresh-water snails in an

aquarium with reference to oxygen; and pill-bugs or cockroaches in a box of chips with reference to contact, all make fruitful laboratory exercises. To be sure the results may not be as uniform as morphological studies upon the same animals *dead*—for the repertory of a living animal is considerably greater than that of a "pickled" one—but, as pointed out in the printed announcement of this conference it is "not uniform but *unified* results" that we are after. Again, the subjects of color in the animal world, of the influence of light upon form and function and the adaptations of animals who love darkness rather than light, may be treated here. The relation of animals to the soil, as, for instance, the development of the burrowing habit, which has occurred in so many diverse types; the relation to air and water as media in which to live and how changes in these media have been effected, are further suggestions for this topic. The isopods to be found under the stones and old boards in almost any locality will furnish excellent material for solving the problem of how certain animals have come to live in the air, while the water beetles of any pond will help to solve the other side of the question of how air dwellers can become adapted to life in the water. It is a valuable and illuminating laboratory exercise to provide each pupil with an active earthworm and set him the problem of making the animal stop moving without killing it, a thing which although not so easy as it looks, can be done by properly manipulating the factors which make up the earthworm's environment. At the close of the period if the pupil has persevered half as much as the earthworm he will be fully as wise as if he had spent the time with a dead worm learning from a laboratory manual that its upper side, whichever that is, is "dorsal," and that the bracelet-like thing up towards its front end is named the "clitellum" with two lls.

Topic 6, The Distribution of Animals, comprises the varied means of locomotion, the effects of sessile life, the migrations of animals with the conquests of new territory and interadjustments in animal communities. Particular species may be found with considerable certainty in definite localities for which fact there are usually good and sufficient reasons. It is worth while looking into the matter, for a laboratory pupil who always has his animals served up to him in glass is not likely to realize this important fact. The intimate examination of the fauna of a very restricted area often discloses the widest horizon. I accidentally discovered this once while teaching biology, if I may

be allowed to illustrate the point from personal experience. A laboratory day had arrived with five large successive classes on hand when at the last moment to my dismay the material, whatever it was, upon which I had depended, proved to be worthless. There was no time to go for more. I happened to have on hand no canned biology that I could substitute. Laboratory work *had* to be done because the program called for it and in desperation I rushed out of the building and seized upon about a square foot of weeds, soil and all, that had escaped the cinders in one corner of the school yard. It was about all the vegetation in sight, by the way, and it was not exactly what would be called an attractive display but it proved a bonanza! In the course of the day we discovered almost everything in that pan of dirt. There were fourteen species of plant life in the first place—each one represented by battle-scarred veterans who had achieved success in a dozen different ways; there were three earthworms, a slug, a carabid beetle, two kinds of caterpillars, a staphylinid beetle, numerous spring tails, a portion of an ant society hard at work trying to repair the damage done by the earthquake, a thriving aphid colony attended by ants upon a pigweed, and under a stone some pill-bugs and a myriapod. What more could one ask?

Not only the distribution of animals upon land but also their distribution in water would find a place here in the scheme of study. Consider that home of life, the ocean, with its various habitats, the strenuous zone between tides, the shallow sea, the deep sea and the surface of the open water. Then fresh water ponds, pools, marshes and streams and finally that region where so many desperate biological problems have been worked out during the ages, the estuaries, which mark the transition region between fresh water and salt. Some of these various habitats are sure to be within the reach of the most denatured city school and a careful census of at least one restricted locality with an analysis of the discoveries made in it is certainly worth while.

TOPIC 7, THE RELATION OF ANIMALS OF THE SAME SPECIES TO EACH OTHER, includes first: the relation of the sexes, sexual selection, secondary sexual characters, etc., and second: the care of the young by those parents who invariably die before their offspring develop, as for example, most insects, as well as those who live to incubate eggs, like the birds or to carry about eggs in safety like the lycosid spiders and crustaceans. The training

of the young by the parents and also lastly the complex relations of communal life are topics to be considered here. Bees or ants can be made to carry on their highly interesting activities in observation houses in the laboratory and it is highly desirable to have as many distractions in the form of living things in the laboratory as possible.

Under Topic 8, The Relation of Animals of Different Species to Each Other, briefly may be considered the struggle for existence and the balance of life among animals. To follow intimately the changes in population from day to day in a standing aquarium which has been stocked with a heterogeneous assortment of pond life is sure to be illuminating. Further subjects for investigation are animals of prey with their adaptations for the capture of prey as well as animals preyed upon with their special devices for defense and safety such as protective coloration, mimicry, armor, etc., and, finally, cases of commensalism, symbiosis and parasitism, with their train of adaptations and modifications as compared with exponents of an independent life.

One of the dangers in such a half year of ecology as outlined above is the liability to become indefinite and confused since there is so much from which to select. Since the object, however, is to develop in the pupil the scientific attitude of the investigator rather than to "salt down" any particular group of facts in his mind, it is the instructor's business to see that each laboratory exercise shall have a definite object and to direct and unify the discoveries of all the pupils. There is no fundamental necessity for a certain inflexible number of laboratory hours each week or for a program so fixed and foreordained that the spirit of investigation shall be robbed of spontaneity. One of the chief joys of biological science is the fact that it contains so much which has not been reduced to certainty and that it has such a large unexplored territory. Here the north pole has not been reached. Why should we be satisfied with a "Cook's tour" through the realms of biology when undiscovered countries awaiting exploration extend to our very doors?

The general subject for the last half year, which fortunately can be treated more briefly than was possible with that for the first half, is

The Comparative Structure and Functions of Organisms.

Here the emphasis is placed upon the *comparative* point of

view. Instead of a somewhat extended study of the entire mechanism of one or more forms it is believed that better results may be attained if one general feature after another of the organic mechanism, as, for example, breathing or reproducing, to be taken up *comparatively* in as many different forms of life as are available. This method, which has long been employed with success in teaching college students the anatomy of vertebrates, seems to me the most logical path to pursue with high school pupils in order to arrive at a comprehensive interpretation of the structure and functions of organisms. Aside from this it is bound to insure a wider acquaintance with forms of life since thus no single animal or plant is selected and laboriously *finished* but rather the entire animate creation, as far as available, is made a foraging ground for illustrative material with the result that repeated encounters with certain forms thus sought out for a definite purpose will be sure to develop in the course of a year a natural familiarity with a considerable number of organisms.

The plan of the last half year has been divided into eight topics which, like those of the first half year, are not of equal importance and consequently will not require equal intervals of time for development.

Topic 1, The Structure and Functions of the Elementary Organisms, includes a brief survey of the Protozoa, the Protophyta and the Protista in general. So far as time and microscopic equipment permit, this fairy-land of science should be explored at first hand by the pupils. It is in any case desirable to develop certain general ideas such as the immense role the unicellular plants play in oceans and other bodies of water in transforming inorganic substances through the energy of sunlight into food available for other organisms, the part they play in the transmission or causation of disease, or the fundamental significance of these unit-like forms in the morphological evolution of the organic world.

Topic 2, The Transition from One-Celled to Many-Celled Organisms, explains itself sufficiently and requires no elaboration here.

Topic 3, The Principle of Division of Labor and Morphological Differentiation, is a general subject which could be worked out in a variety of ways—preferably with the lower forms where there are fewer complications. The object of this topic is to give a logical explanation for the diversity of forms

among organisms and to furnish a general key for interpreting the organic adaptations, the study of which constitutes the chief lure to the biological student.

Topic 4, Individuals and Colonies, is another general subject of rather minor importance which nevertheless is well worth a brief treatment.

Topic 5, The Structure of Many-Celled Plants, deals with the comparative anatomy of plants. The following list of sub-topics will make sufficiently clear the manner of treatment recommended for covering this rather familiar field of biology.

1. Cells, tissues and organs in general.
2. Protective organs.
3. Supporting organs.
4. Organs of nutrition.
5. Organs of respiration.
6. Organs of transpiration and conduction.
7. Organs of response to stimuli.
8. Organs of reproduction.

This to be followed by Topic 6, The Functions of Plants, under which is considered, first: their metabolism, second: their movements or reactions to stimuli, such as geotropism, heliotropism, thigmotropism and the like. It is not intended to divorce structure and function as the arrangement of the matter under two topics might indicate. It would be absurd to try to separate them or to discuss which is the more important and the fundamental danger in limiting or even in introducing the study of an organism with the dead animal or plant as has so often been done, lies in the fact that the structure and function are thus made separate things. The most important fact about an organism is that it lives—consequently the logical avenue of approach in studying it is not, "Here is a dead mechanism. What could it do when it was alive?"—but rather: "Here is a living object. What is the mechanism by which it acts?"

Topic 7, The Many-Celled Animals, is designed to transfer attention to the animal world in particular without attempting a separate treatment of structure and function. While cell-differentiation, and the germ-layers and tissues might naturally be developed from the morphological point of view, the organs and their functions would best be treated comparatively, being illustrated from the evolutionary standpoint with as wide a range of animals as possible. A brief scheme in detail takes up, 1.

Protective organs, as typified particularly by the integument; 2. Locomotor organs, such as skeletal structures and muscles; 3. Organs of metabolism, that is, the digestive, circulatory, excretory and respiratory organs; 4. Organs of reproduction, and 5. Organs of sensation and correlation, as, for example, the central nervous system, the transmission systems and the sense organs.

The final topic suggested, No. 8, is MAN'S PLACE IN NATURE. This topic includes, first: an orientation of the pupil with respect to the theory of descent, that is, the evidences for evolution from classification, anatomy, fossils, embryology, distribution and experimental breeding; Lamarck's and Darwin's explanations of how evolution has come about and a clearing up of the present day standpoint concerning this great epic of biological thought. I am firmly convinced that pupils of high school age are ready to appreciate much of this majestic canvas if only it is properly unrolled before their eyes. In its main lines it is a story that when simply told is sure to grip the imagination, illuminating and unifying the entire year's work. A second sub-topic, introduced here, is the natural history of man as shown first by what we know of his prehistoric days and his particular ascent from humble origins to his present high estate, and second: by the examination of the existing races of mankind, their distribution over the earth and their conquests of the controlling forces of nature. Finally, has not the time come when we can conclude an ideal course in secondary biology with something of a practical moral in the way of elementary eugenics? Some sort of correct knowledge of the remorseless laws of heredity as applied to man and of the immense possibilities in the hands of those who understand how to control those laws, is of the highest importance to every future citizen. The time is surely coming when the scientific breeding of the human species will be at least worthy of as much attention as the breeding of cows or cabbages, which for a long time have been objects of human solicitude.

It will be seen that throughout this entire scheme of study no definite portion of time has been set aside for the identification and classification of animals and plants. It is assumed that the preceding nature-study in the grades has accomplished a general working vocabulary of the names of living things, not scientific names nor technical classifications, whose acquisition belongs

largely to the advanced work of the college or university, but common usable names. If this scheme of study has any virtue at all it is pretty sure to develop unconsciously and as a by-product, a working plan of classification and identification sufficient for all practical purposes. It is a mistake to advance the names of things before the pupil sees any sense in them. Too often learning the scientific name of an animal so exhausts the energy and dulls the interest that the pupil has no desire to go further.

To summarize: the proposed ideal course in high school biology demands (1) a study of organisms in their living relations rather than a morphological inquest upon their dead remains; (2) less of classification and identification of species as an end in itself; (3) a more comparative study of structure and function than the ordinary type-study method insures; and (4) development in the pupil of a greater power and independence in interpreting the living world than seems to be possible through laboratory directions so detailed and complete that he is robbed of the initiative which it should be the instructor's attempt to foster.

One of the largest deposits of zinc in North America is that known as the "Big Ledge," on the upper face of a mountain above Arrow Lake, British Columbia. The ore is in the nature of "black-jack," averaging 30% to 40% zinc. The deposit has been traced over three miles, with a width of 50 ft. to 100 ft. So far no development work of importance has been done on it.

In Scotland and New Zealand, where there are no known petroleum deposits, a remarkable substitute has been found in the existence of immense deposits of bituminous shales. These have been milled and distilled for oil, and for a great number of secondary products. The works have been running for many years in both countries, and a thriving business has been established and many millions added to the public wealth.

The iron ores which have been mined in Cuba up to the present time consist largely of hematite and magnetite, and are obtained near Santiago, in the province of Oriente (Santiago). Recently large deposits of brown ore have been attracting considerable attention, especially those of the Mayari and Moa fields in Oriente Province and those of the Cubitas field in Camaguey Province. Some 819,434 tons of iron ore were shipped from the mines in 1908, the United States taking 579,668 tons.

SCIENTIFIC AND MATHEMATICAL TEACHING IN THE FUTURE.

By Fredus N. Peters,
Kansas City, Mo.

(*Continued from the November issue.*)

Physics is the spectacular science. Almost every year sees some discovery which causes the world to stand aghast and wonder what the physicist will do next. Even yet we have not recovered from the effect upon us of the recent practical applications of the wireless telegraph. We can hear even yet the snap of the instruments on that ill-fated ship, sending out in all directions appeals for help; we can see the tapes reeling off the messages at the various receiving instruments; we can almost hear the orders of the captains as they bid their pilots make all haste toward the scene of the disaster. It seems to me no more sublime, no more faith-inspiring object-lesson ever presented itself to human intelligence than that of these ships responding to the call of the sinking *Republic*. And as we see in the blackness of that stormy midnight the various searchlights of the rapidly approaching rescuing ships, we behold a picture immeasurably surpassing anything the most fanciful Grecian mythologist ever dared to dream.

Many years ago Lister declared that it is possible to wipe from the face of the earth every contagious disease; yet we are only just beginning to make any effort in this direction. Every year sixteen to eighteen out of every ten thousand die of tuberculosis and yet, unless this dreadful white plague happens to seize someone near and dear to us we think little of it. We should be thankful that now there seems to be a considerable degree of interest among those best able to handle the situation—the physicians—and it is to be hoped that much good will come of it. Dr. Latham, a prominent physician and specialist of London, recently predicted that in thirty or forty years tuberculosis will be as completely a disease of the past as leprosy or cholera now is. Sixty years ago smallpox caused a large proportion of the deaths in England; now it is very seldom even a serious disease. Cholera has lost its terrors; malaria and yellow fever are understood and practically under control; so we may hope that the prediction of the London physician may come true.

But if it does it must be through the education of the masses, and much responsibility rests upon the teacher of science.

The physiographer likewise will be called upon to assist in working out the great economic problems of life. As old and now known deposits of mineral become exhausted the geologist by his knowledge of the earth's crust will be of great assistance in leading to other supplies and will be called upon in various ways.

At the same time much of the work required of science in carrying forward the great enterprises of the world will need the assistance of mathematics. Neither chemistry nor physics can do much without a broad knowledge of mathematics; none of the great engineering problems, absolutely essential for the future, can proceed at all without mathematics. In short, the future economic progress of the world depends largely upon the united efforts of the twin sisters, mathematics and science, and hand in hand they must go down the centuries together.

Admitting that these statements are true, what have they to do with us as teachers of mathematics and science? I have taken as much time as was allotted me in stating my case and must leave the application largely to you. Time was when it was considered advisable and profitable for scholars to spend their time in discussion of such subjects as "How many angels can stand on the point of a needle?" or "Whether angels in going from one place to another pass through intermediate space." Nor have the various schools of philosophy been the only ones to waste time and energy in useless investigations. Chemists of great note have spent months in discussions as to the linkage of the carbon atoms in various compounds: for instance in the benzene series, Kekulè, Claus, Korner, Ladenburg and others spent months in advocating their own ideas and opposing those of their contemporaries when in reality in essentials several of them were the same.

Scientific theories usually are but scaffolds: as long as they are a help to carry on our work to greater heights, they are valuable and necessary; when they have served their purpose let them be torn away.

I should not attempt to maintain, however, that theoretical considerations may not sometimes be of great value; for occasionally they have led to remarkable discoveries. If a process of converting starch into caoutchouc is ever perfected it will undoubtedly come through a very complete knowledge of the con-

stitution of such compounds. Nevertheless it is very easy to waste valuable time in such study and discussions.

The day is near at hand when the very practical age in which we are living is going to demand of our science and mathematical teaching something of a practical character. Not only must the high schools which will give the great majority of our students about all the science and mathematics they will ever receive, emphasize the practical applications to everyday life, but the universities will feel the same demands upon them. There will always be some, and for that we should feel profoundly thankful, who will take delight in the mental gymnastics incident to the most abstruse theoretical scientific research; but the great majority should receive a different training. Feeling this demand already upon them, some of our universities have succeeded in establishing fellowships supported mainly by the lines of industry to be profited by the results of the research work, which are destined to be of untold value to our citizenship. To illustrate, one of our sister middle western universities, among a number of others, is engaged upon the following problems:

1. An investigation into the chemistry of laundering, having for its object the saving of the people's linen.

2. A search for a new diastase, which at present is expensive. The investigation has as a matter of fact developed into an attempt to make a new fodder upon scientific principles.

3. An attempt to utilize the constituents of waste buttermilk which at present in butter factories goes down the drain.

4. An investigation into the chemistry of baking.

5. An attempt to improve the enamel upon the enamel-lined steel tanks used in all kinds of chemical operations on a large scale.

6. An investigation into the relation between the optical properties of glass and its chemical composition.

7. The discovery of new utilities for portland cement and of improvements in its manufacture.

8. The discovery of new uses for ozone.

9. An investigation into certain packing house problems.

In reality, all of our great universities, especially those of the West, are becoming great centers of investigation along practical lines, and by their influence are helping us who are laboring in a more modest way to carry the light to those farthest from its source.

America, with its great natural resources, and protected by a tariff in many ways, has never appreciated the help that science can give. England has long been regarded, in a commercial and industrial sense, the foremost nation of the world. But Germany in the last quarter of a century has come near outstripping her great rival. Why? Simply because of her vast army of trained scientists who are sent out every year with research powers trained to the utmost. It is by these men that processes are discovered for making thousands of manufactured articles just a little more marketable than those of any other nation—by these men, that the fields once used to grow maddar and indigo can now be utilized for foodstuffs, while at the same time the most beautiful dyes are at the command of even the poorest peasant. It is by these same men that the soils of the fields have been tested and made to yield their best—by these men that almost every phase of domestic and industrial life has been enriched, and the German nation placed upon a footing where she can meet the world with such aggressiveness.

It is said that a single anilin factory in Berlin employs continually about one hundred and fifty chemists. This is but an example of hundreds of manufacturing concerns all over the empire. Here in America, such a large company as the Proctor and Gamble Soap Factory at its Kansas City branch has one head chemist with two assistants, one of these being little more than a bottle washer. Armour's, with the hundred or more by-products of the packing industry to be refined, improved and tested, has but two.

But this day is nearly past; and I believe firmly that we, as teachers of mathematics and science, will be called upon to meet this demand in a practical way.

At the height of the mad frenzy of the French Revolution, Lavoisier, the greatest chemist of his time, the first to perceive the truth as to the nature of combustion—the man who was compelled to maintain his position against the most of the remainder of the scientific world, gained the ill-will of the leaders in that terrible maelstrom. When his friends pled for his life on the grounds of his great scientific attainments and what he could do for the world, his enemies shouted, "Away with him; we have no need of chemists," and snuffed out his life as they would have the flame of a candle. Some time ago the *Wall Street Journal,* which I need not say is not a scientific publication, in

an editorial under the heading "Science as a Financial Asset," used these words: "Every commercial transaction in the civilized world is based on the chemist's certificate as to the fineness of gold, which forms our ultimate measure of value. Faith may remove mountains but *modern* society relies on dynamite. Without explosives our great engineering works must cease and the Panama Canal no less than modern warfare become impossible. Chemistry has made possible the transportation systems which span the leading countries of the world. It has made it possible to turn to man's service the wealth of the mineral world. By analysis of plants and soils, the waste materials of the world have been brought to the growing of crops. Indeed every great industry, whether it be farming, manufacturing, transportation or mining would almost immediately relapse to barbarism if the secrets of the chemist and physicist, the geologist and mineralogist, could be gathered up and cast into the sea."

In conclusion, let me say that what I have attempted to show is, that the United States is rapidly approaching a period in her history when very different methods will have to be employed along a large number of lines: when slipshod and easygoing practices will not suffice. These changes involve problems of momentous consequence which must be solved largely by those skilled in mathematics and science. As it has been the public school which has taken the miscellaneous foreign population poured unceasingly upon us and in a generation made them into patriotic American citizens, so it will be the high schools and universities which will care for these new difficulties. *We* must furnish the men fitted to grapple with the problems which will soon confront us. To do this means that our teaching must partake eminently of the practical. If we heed the cry now but faintly coming to us from all quarters, we as teachers of science and mathematics shall be largely instrumental in working out the destiny of this great country; and though now we may be as Newton in his great modesty once said but children playing upon the seashore, picking up now and then a more beautiful pebble or a rarer shell than common, while the great ocean of truth lies all undiscovered beyond, may we not hope that if we hearken conscientiously to the demands upon us we may be largely instrumental in directing others who may come after us safely over that great ocean of truth or in learning much more of its wonderful mystery?

A NEW FORM OF EWING'S MODEL OF "MOLECULAR MAGNETS" FOR PROJECTION.

By Will C. Baker,

School of Mining, Queen's University, Kingston, Ont., Canada.

Having had experience with several forms of Ewing's molecular magnet model[1] the writer has devised one that is most satisfactory both in simplicity of construction and in ease of manipulation. It may be used either in a lantern for projection, or on a laboratory table for direct observation. The numbers given refer to a model used chiefly in a lantern with a four inch condenser.

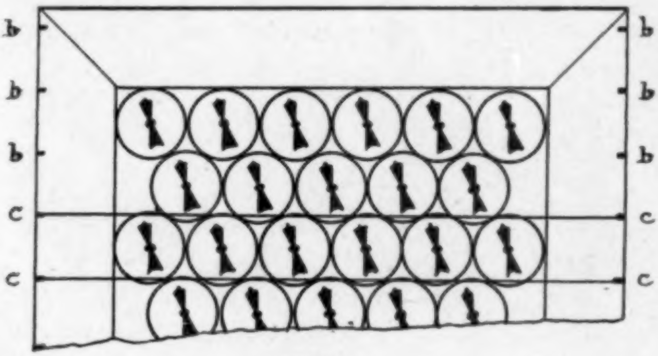


Fig. 1.

Thirty-three small compasses of the sort used for plotting magnetic fields[2] are packed in a wooden frame between two sheets of glass as shown in plan and section in Figures 1 and 2. In Figure 2 the glass tops of the compass boxes are hatched vertically and the top and bottom glasses of the frame are hatched at sixty degrees. The magnetic needles in Figure 2 are, for clearness of depiction, shown in a line across the frame, a different azimuth from that used in Figure 1. The upper glass sheet is held in place by strips of wood with brass pins, as at aa, Figure 2. Brass brads are used in the construction of the frame itself. Along the edges of the frame, as shown at b, b, b, Figure 1, are a series of shallow saw cuts to receive the wires of the solenoids. Two of these wires are shown at cc, Figure 1.

[1]Magnetic Induction in Iron and Other Metals. J. H. Ewing.

[2]The needles are one half inch long, pivoted between two glass discs eleven sixteenths inch in diameter, held together by a band of brass. See figures.

A sufficient length of No. 30 d. c. c. wire is doubled on itself and then twisted. Starting from one end, this double wire is wound into the saw cuts, being looped through itself, in a sort of "button-hole stitch" at each turn (See Figure 3). On reaching the end of the frame the wire is wound back in the same way and the ends soldered to four binding posts. The result is

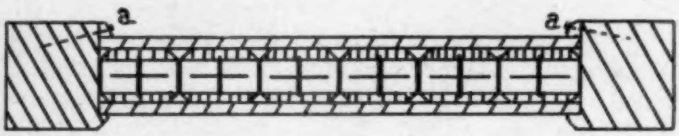


Fig. 2.

two independent solenoids of two layers each; one of these serves to carry a steady current to neutralize the earth's magnetic field, and the other connected to a source of potential through a reversing switch and rheostat, produces the variable field necessary for the experiment.

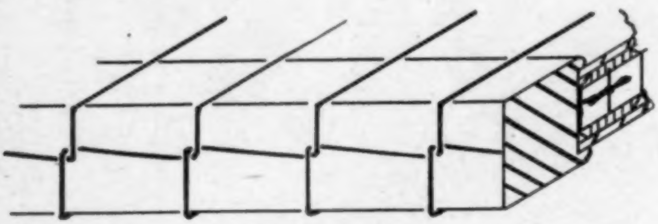


Fig. 3.

The adjustment of the compensating current is easily made as follows: The frame is set with two edges parallel to the lines of the earth's horizontal intensity and a bar magnet is moved above the small magnets to set them in motion. On the removal of the bar magnet the needles settle to rest, and if the compensating current is too strong most of them will point south; if the current is too weak most of them will point north; if the current is just sufficient to produce compensation the needles on settling to rest from a disturbance will form themselves into beautiful closed curves. From this point all the effects shown by the more complex and expensive models are easily obtained.

For direct observation the compasses are viewed through the wires of the solenoids, but in the projection lantern the wires are not seen, for the effect of the bevelled edges of the compass boxes broadens out the shadows of the box edges and completely hides them. In handling there is no trouble from the magnets

being shaken off the pivots, as in most forms; and the system may be put into the hands of students without fear of damage. The solenoids, too, are far more compact than the Helmholtz arrangement usually employed in projection apparatus. The field is also more uniform and the currents required much smaller.

Of course as with any type of the Ewing model slight trouble may be experienced from a permanent magnetization of the iron in the lantern producing an uneven field in the apparatus. For this reason the writer prefers to build up a lantern by enclosing the arc lamp in a box and then to project vertically through a condenser and objective clamped to a brass rod. The extra trouble is easily repaid by the improved result.

For ease of construction and manipulation, for solidity and for good results the simple form described above gives more satisfaction than any other arrangement known to the writer.

RANDOM SUGGESTIONS FROM ST. LOUIS.

The Discussion of the Laboratory Experiment.

By Charles H. Slater,

William McKinley High School, St. Louis, Mo.

The discussion of the experiment is somewhat of a dilemma to many of our pupils. With few exceptions physics teachers agree that the discussion is second only to the experiment itself. Also with few exceptions the pupil asks, "What shall I discuss?" and "How shall I discuss it?"

These questions are often answered orally as they arise. Some pointers or suggestions are frequently given in the laboratory manuals in use; for example, the following is often found: "Do your observations and results confirm the deductions of the theoretical discussion above? Make a statement as to what has been deduced and verified." The average pupil would not be helped very much by this pointer. (?) He does not understand how to show that his data verifies the truth or law studied. When told to "derive the law or to state the conclusions drawn from the data," we find a verbatim text-book statement of the law or relation under consideration. Pointed questions are frequently put in to direct the pupil's manipulations but they do not aid him very much in his discussion of the experiment. Are not the use and the interpretation of data

equally as difficult for the average pupil as the manipulation of apparatus? Are they not of equal importance? Suggestions and questions should also be given to assist him in this part of his work.

The pupil beginning the study of physical science is expected to form conclusions and generalizations which required more than one and one half hours of careful work on the part of greater scientists than himself. Is it not asking a little too much of the *beginner* that he derive laws or understand their verification in the short time allotted without much directive assistance? And will it not require specific, instead of general, suggestions or questions? The experiments cover too wide a range of topics and are too varied in their character to be discussed in the same way from a few general pointers. In the same division of the subject some experiments admit of very accurate quantitative work, others are wholly qualitative; a few are attacked geometrically while many involve more or less computations; some are to establish definitions and known constants, while many study relations between variable quantities; some are to develop general principles and others are applications of these principles. The necessity of definite direction in the discussion of each experiment seems obvious.

Since then the average pupil of the tenth and eleventh grades cannot always be expected to see the connection readily between his data and the conclusions sought, we have been working on the following plan in the St. Louis high schools. Our pupils receive with the printed directions for each experiment a short set of questions based entirely on their work and data. These are intended to direct the pupil in the "discussion" part of the "write up" of the experiment which they prepare at home. These questions cannot be answered by verbatim statements from the text, but a review of the work and a reinspection of the data is necessary. We endeavor to so state the questions that the pupil must use his data in some way in answering them to bring out the point of the experiment. The main object of the experiment therefore is not the getting of the data, but its *use* in attaining the purpose of the experiment. The data, then, is not an end sough but a means to the end. The conclusions follow from the data and observations, and not from the text.

We are using this plan with second and third year pupils with better results we think than we secured before. Many pupils said at first, "It takes more time but we understand it better."

The discussion questions may be added to or varied as, in the opinion of the teacher, the work of the class seems to demand. The pupil gets help at the time when he needs it and of such a nature as to be of much greater value than the verbatim statement of portions of the text. He is therefore more able to reason to conclusions through his own observations than to jump to one that he copies from his book.

APPARATUS FOR STUDYING THE FIELD OF A SWINGING SOLENOID.

Description.

The apparatus consists of a light wooden ring, 2½ inches in diameter, and about ¾ of an inch wide, around which are wound 6 or 8 layers of fine insulated copper wire, No. 36. Of course the more turns the more sensitive the apparatus. The ring is suspended by the same wire, the ends of which are extended beyond the coil for 6 or 8 feet. This wire will stand the current through one 16 c. p. lamp at 110 volts, that is about ½ ampere, for a few minutes at a time, without getting too hot.

Uses of the Apparatus.

First.—If the ring is suspended so that the plane is north and south, and the current is sent through, it will turn in the earth's field. If the current is reversed, it will turn in the opposite direction.

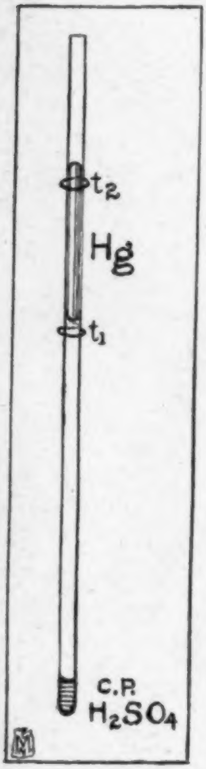
Second.—If placed between 2 bar magnets, unlike poles facing the ring, and the plane of the magnets and ring coinciding, and the current is turned on, the ring illustrates the D'Arsonval effect by taking a position at right angles to the plane of the magnets. Again, reversing the current causes the ring to rotate in an opposite direction.

Third.—The behavior of the ring, when carrying the current, and brought near a magnetic pole, is most striking. Thus, with a given direction of current, it is repelled, when a north pole is brought near one face of the ring, and attracted, when the same pole is brought near the other face.

If the side which was repelled is placed over the pole while the current is off, and the current is turned on again, the ring will be thrown off, will reverse, and return to the magnetic pole. If a powerful electro-magnic is used, the effect is strong enough to toss the ring into the air when the current is turned on.

APPARENT COEFFICIENT OF EXPANSION OF AIR.

Various methods have been suggested for this study of Charles' Law. One that is easy of manipulation is suggested by the accompanying figure. It is a combination of two methods found in various texts. Upon trying it without the acid we obtained results much too large and quite erratic. The use of C. P. sulphuric acid in the lower end of the tube to insure dry air improved our results very greatly.



It did not prove a difficult or long task to make 35 tubes. We use small tubes, 35 cm. long and from .5 mm. to 1 mm. diameter. They must be thoroughly cleaned and dried. An open end is then touched to the acid a moment and capillarity causes enough to enter the tube. This same end is then closed in the gas flame and by a quick throw or flit centrifugal force drives the acid into the tip displacing any air bubbles lodged there. The thread of mercury, several centimeters long, is inserted with a long, slender capillary pipette, easily made from a larger tube drawn out in the bunsen burner flame. This is best done with the tube in a nearly horizontal position. At the same time the thread of mercury can be easily adjusted along the tube to secure a convenient length of air column. Hydrometer jars of ice water and hot water, and a tall steam boiler give suitable temperature controls. Threads t_1 and t_2 tied about the tube serve as indices and may be adjusted while it is in the temperature control. It is then removed and l_1 and l_2, the lengths cold and hot, measured. The apparent coefficient of expansion then equals $\frac{l_2 - l_1}{t_2 \, l_1}$. The average result of 60 pupils' work recently on the above was .00364, and 55 were within five per cent of the correct value.

POLARIZATION OF THE VOLTAIC CELL.

We have not found the usual laboratory method for showing the above at all satisfactory. We have varied the resistance in circuit, the strength of the solution and have used several compass galvanoscopes but to no avail. In all cases when the current is strong enough to give an average deflection of 50° to 60° the cell would polarize before the needle stopped swinging at about 30°, and the pupil could see almost no drop in the following readings. We have not found a compass galvanoscope sufficiently "dead beat" to overcome the difficulty—but have hit upon the following device which gives very satisfactory results. We set up the cell as usual with acid 1 to 25 parts water, an amalgamated zinc, preferably a roughened copper strip, a 10 turn tangent galvanoscope with enough resistance to give 25° to 30° deflection when polarized which occurs before the needle stops swinging. If now the plates are lifted out and the copper strip lowered into a strong solution of copper sulphate a moment and then replaced in the acid the needle is found to stop swinging at about 60°. In a half minute or so it is seen to drop slowly back to about 30° again. When this is observed the pupil is directed to look to the surface of the copper plate; minute bubbles are just beginning to become visible over its surface. A few repetitions soon satisfy the pupil as to the cause and effect of polarization; he also sees a suggestion as to the remedy in the use of the copper sulphate. The film of sulphate clinging to the copper plate holds polarization back long enough to allow the needle to become steady, and also to enable the pupil to observe the drop in deflection together with the formation of bubbles on the copper plate. He is now ready to take up the study of the Daniell Cell.

Mineral paints may be divided into three groups—(1) natural products, such as ochre, umber, sienna, hematite, siderite, limonite, ground slate and shale, which, after mechanical treatment, such as cleaning and grinding, are either used directly as pigments or are first roasted to give certain desired colors; (2) chemical products, such as zinc oxide, leaded zinc oxide, zinc-lead, sublimed white lead, and sublimed blue lead, which are made directly from ores of valuable metals; and (3) chemical products such as basic carbonate white lead, litharge, red lead, orange mineral, lithopone and Venetian red, which pass through several metallurgical and chemical processes in their preparation from the original ores.

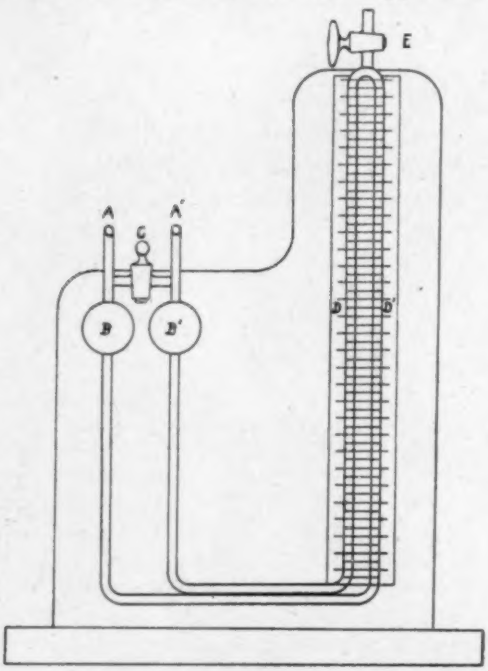
A THERMOSCOPE AND ITS USES.

E. J. RENDTORFF,
Lake Forest Academy, Lake Forest, Ill.

Every chemical reaction and many physical phenomena are accompanied by thermal changes that are essential to the transformation and conservation of energy. These heat reactions are too frequently not given the consideration that their importance demands. The reason for this probably lies in the difficulty generally experienced in making these thermal changes apparent to a class, especially in a large lecture room.

I wish to present a type of thermoscope that can be employed in illustrations of this nature and give a brief outline of a few essential experiments that are within the grasp of secondary school students. Some of these are fundamental and should be illustrated in every preparatory course. The list is incomplete and can be extended and often modified by any experienced teacher.

There is no instrument at any price that has a greater range of usefulness than the thermoscope. It is, without exception, the most essential of all our lecture room apparatus.



The thermoscope consists of two bulbs, B and B′, connected at the bottom to the tubes D and D′, and on top to the stopcock C. At A and A′ rubber tubes are attached which lead to accessory bulbs. The bulbs B and B′ and their lower attached tubes are half filled with some colored liquid that acts as an indicator. A glass scale is placed behind the tubes D and D′, so that the exact position of the indicator can be noticed.

When the stopcock E is open either branch of the thermoscope can be used as an independent instrument. When closed the two branches act in opposition. If the same pressure is exerted at A and A′, the indicator will rise slightly in both tubes, but always to equal heights. Two sources of heat can thus be balanced, either of which would force the indicator beyond the upper bend of the tubes D and D′.

If one branch only is used a momentary opening of the stopcock C will keep the indicator at a convenient position on the glass scale. The instrument is therefore entirely under control.

Two separate experiments can be performed at the same time if the stopcock E is open, C closed and two independent accessory bulbs attached to A and A′.

The accessory bulbs are the seat of the various thermal activities. They are of various shapes, as illustrated in Figure 2.

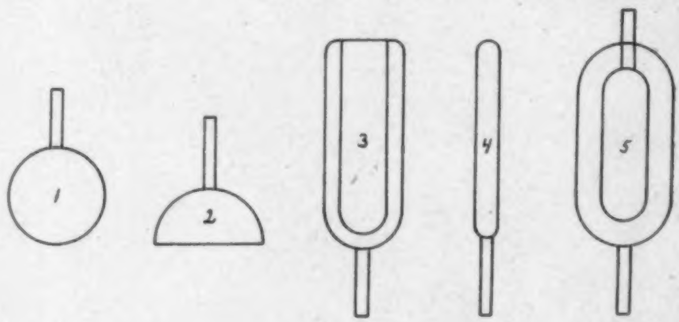


Fig. 2.

If any bulb, say (1), be attached to the tube A of the thermoscope and heated the expending gas forces the liquid down in tube D and up in D′. The distance between D and D′ is a measure of the amount of heat applied.

The thermoscope can be placed in front of a projection lantern and used in a lecture room of any size. The accessory bulbs are held in convenient standards removed from the zone

of heat produced by the lantern. In this way the heat absorbed by the evaporation of one drop of water will make the projected image rise several feet.

The appended list of experiments illustrate the manifold uses of the thermoscope:

MECHANICS.

1. Pascal's Law.

Connect two legs of a T tube to the two branches of the thermoscope. Blow through the third leg. The two indicators will rise to the same height.

2. Laws of Fluid Pressure.

Take several glass tubes of the same internal diameter, whose ends are bent in different directions. Connect them alternately to one branch of the thermoscope and immerse to different depths in various liquids. Prove that:

(a) The pressure varies as the depth.
(b) The pressure is independent of direction.
(c) The pressure varies as the density of the liquid.

3. Induced Air Currents.

Construct an atomizer and connect the thermoscope to the tube that ordinarily projects into the liquid to be sprayed. Blow through the other tube and note the lowering of the indicator.

4. Adhesion.

Close the upper stopcock. Connect a capillary tube, containing some water, to one branch of the thermoscope. Blow through the other branch. The indicator is elevated quite a distance before the adhesion of the water and glass is overcome.

5. Surface Tension.

Attach a T tube to one branch of the thermoscope. Dip one leg of the tube into a soap solution and blow through the third. The indicator will rise several cm. If the third tube is held closed a very slight elevation still continues, due to the surface tension of the bubble.

6. Effusion of Gases through Porous Substances.

Put a cork in the opening of an unglazed earthenware vessel and connect with the thermoscope. Cover the vessel with a bell jar and fill with hydrogen. Note the elevation of the indicator.

7. Effusion Diminishes with the Density of the Gas.

Repeat Experiment 6 with illuminating gas and note the diminished elevation.

8. Siphon.

Construct a siphon with an open tube at its bend. Fill it en-

tirely with water and hold a finger over the upper tube. Connect either end of the siphon to one branch of the thermoscope, holding the rubber tube horizontally and place the other leg of the siphon in a beaker of water. The longer tube exerts the greater pressure. If the length of the longer leg be increased by lowering the rubber tube this difference of pressure becomes larger.

9. Specific Gravity of Liquids.

Attach a straight glass tube to the thermoscope. Take the zero reading of the indicator and then thrust the tube a measured distance (about 10 cm) into a tt filled with kerosene. Note the exact elevation of the indicator. Repeat for water.

$$\text{The sp. gr. of the kerosene} = \frac{\text{Rise of indicator for kerosene.}}{\text{Rise of indicator for water.}}$$

This experiment can be modified in several ways.

Expansion and Compression.

10. Expansion of Gases by Heat.

Attach (1) and heat by holding it in the hand.

11. Cooling of a Gas by Expansion.

Connect the inner bulb of (5) to the thermoscope, and the outer one to an air pump. Exhaust the air and note the lowering of the indicator.

12. Heating of a Gas by Compression.

With the apparatus attached as in Experiment 11, allow air to enter the outer bulb. Heating occurs at once.

13. A Hot Liquid Has Less Density and Consequently Floats on a Colder One.

Attach both (1). Fill the lower half of a hydrometer jar with cold and the upper part with warm water. Immerse the bulbs, holding one near the bottom and the other near the top of the water.

14. Anamalous Expansion.

Adjust the apparatus as in Ex. 13. Fill the jar with water at about 8° C. and hold a large piece of ice near the middle of the jar. The bottom first cools to 4° C., the top next to 0° C., and finally the bottom to 0° C.

15. Convention Currents in Air.

Attach (2) and hold alternately at the same distance above, below and on the sides of a hot solid. The heating on top will be much greater than in the other positions, due to the upward convention of the heated air.

(*To be continued.*)

MASS AND INERTIA.

By Henry Crew,
Northwestern University.

[The following was written by Dr. Crew in answer to the question asked in the department of Science Questions, "What is the distinction between mass and inertia?"]

Matter is something which presents itself to our senses under an almost endless variety of aspects. Certain of these aspects are considered by chemists; others are abstracted and considered by geologists; still others are studied by physicists. Each special science, indeed, segregates in thought, not certain *materials,* but certain *aspects,* in the world of matter as its own peculiar field of investigation. These delimitations having been tacitly agreed upon form the boundaries of modern science, in so far at least as these sciences can be said to have any boundaries.

Physics, in this manner, sets certain definite limitations to its inquiries and adopts certain conventions which have proven most helpful. Among these world-wide agreements the following refer to the question recently raised in School Science and Mathematics.

(1) It is agreed that the physicist will leave to the metaphysician all discussion of the nature of "substance."

(2) It is agreed that we shall, for all practical operations of the laboratory and for all purposes of computation, adopt the one property of inertia (which all bodies exhibit) as the measure of the "amount of matter." Were all bodies of equal density volume would doubtless have been adopted as the measure of the amount of matter.

(3) It is agreed that we know nothing, and can probably learn nothing, about the ultimate nature of matter. We have, therefore, no method of determining absolutely the quantity of matter in any body. But abstracting, in our thought at least, the inertia of matter from the infinitude of other properties which it presents, we find that we can easily compare the inertia of one body with that of another; in other words, we find that inertia is a measurable quantity.

(4) It is agreed that we adopt as our unit of inertia either that of a certain body in Sevres called the Kilogram des Archives, or that of a certain body in London called the Standard Pound.

(5) It is agreed that we shall call the number of pounds or kilograms of inertia in any body its "mass." In other words, we agree in the physical sciences to use "quantity of matter," "quantity of inertia," and "mass" as exact synonyms, understanding that, in the broad sense of the term, we know nothing about the quantity of matter, and that inertia, in its wide sense, denotes merely a quality of matter; namely, a certain "laziness" in stopping and starting.

ABSOLUTE TEMPERATURE.

BY K. E. GUTHE,
The University of Michigan.

[The following was written by Dr. Guthe in answer to a question in the department of Science Questions.]

"What is absolute temperature?"

The term "absolute temperature" as used in most text-books is open to criticism. The name implies that temperature measured in the absolute scale should be independent of any particular thermometric substance. There is only one scale which fulfills this requirement, namely the thermodynamic scale of temperature, based upon the efficiency of a reversible steam engine. This efficiency does not depend upon the nature of the working substance, but only upon the temperatures between which the substance works. The "absolute zero" is the temperature to which the working substance must be cooled in a Carnot cycle so that the efficiency of the engine becomes unity, or, so that all heating entering the engine is converted into work.

Absolute temperature is therefore temperature measured by the thermodynamic scale.

This scale agrees for all practical purposes with the indications of the gas scale if we choose a proper zero point.

The temperature which is usually called the absolute zero, but which I prefer to call the "zero point of the gas scale" is that temperature at which hydrogen gas would exert no pressure, supposing that it remain a gas even at the lowest temperatures.

This temperature is easily found from the equation

$$P_t = P_0 (1 + \alpha t)$$

if we place $P_t = 0$.

Frequently the zero point of the gas scale is defined by making use of change of volume of a gas with temperature. But this is contrary to international agreement according to which

the standard thermometer is a constant volume hydrogen thermometer and temperature differences are measured by differences in pressure.

It is difficult to propose a new name for temperatures measured from the zero point of the gas scale. I have been searching for one, but without success. "Hydrogen temperature" is a rather clumsy name and does not express exactly what we want. Here is a chance for a philological genius; until he arises we shall probably continue calling these temperatures absolute temperatures.

The numerical values of a temperature expressed in the absolute and in the hydrogen scale are very nearly the same, differing by less than 0°.001 C. between —50° and +150° C. and amount only to 0°.044 C. at +1,000° C. However, if we wish to formulate an "exact" definition we must distinguish between them. It is a pity that nature has not supplied us with a perfect gas, for, if this were used as thermometric substance the two temperatures would become identical and no conscientious scruples would arise when we *speak* of absolute temperatures, but *really mean* temperatures measured from the zero point of the gas scale.

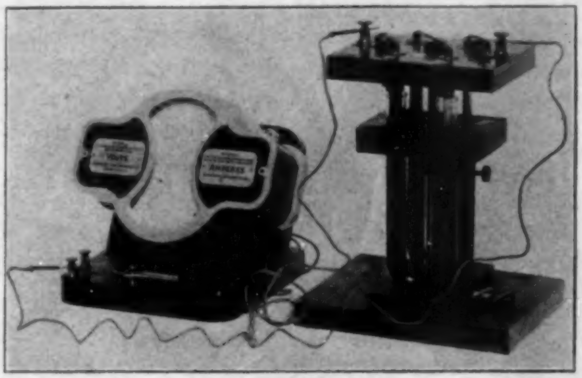
A DEVICE FOR SHOWING THE EFFECT UPON E. M. F. AND INTERNAL RESISTANCE OF ARRANGING TWO SIMILAR CELLS IN SERIES AND PARALLEL.

By Lewis C. Williams,
Erasmus Hall High School, Brooklyn, New York City.

To realize nearly ideal conditions a cell is needed whose E. M. F. shall be very constant and whose internal resistance shall be large enough to enable one to neglect the small external resistance offered by the ammeter and its leads but shall be low compared with that of the voltmeter. With the resistance of ammeter and leads amounting to 0.2 Ohm and that of the voltmeter about 1,000 Ohms the best value for the internal resistance of the cell has been found to be from 12 to 15 Ohms. No cell in common use fulfills these conditions.

By the aid of the cut the device is seen to consist of a baseboard 7"x9", in the center of which is fixed a brass tube terminating at its upper end in the upper surface of the middle board. This middle board is cut into from both sides providing openings to receive two U tubes 8" long and ⅞" in diameter.

These are thus held in position, side by side, and about two inches apart, each being secured by a wooden button. A brass rod about 10″ long slides in the brass tube mentioned above and



passes upward into the top board which it supports. This rod is grooved longitudinally to receive a set screw to be seen at the right, just below the middle board, and thus furnishes the plunge feature of the common Grenet cell. Through the top board pass downward four brass thimbles directly over the four arms of the U tubes. These thimbles are about 2″ in length with a flange at the top to hold them in position and with set screws at their lower ends to secure the carbon and zinc rods. These rods are about 5″ long and are inserted from below. On the top of each thimble and on each brass strip holding a binding post is soldered a copper cup for mercury, six in all. Into these cups are placed the ends of copper spanners to make the connections desired. The liquid used is the stock solution of the common Grenet cell.

To secure similar cells, i. e., cells of the same resistance, the rods are adjusted to the proper height and then upon lowering the top board the apparatus is ready to use.

The above device is quickly and easily adjusted, positive in action, gives uniformly good results and possesses the necessary fool-proof features. Below will be found sample data hurriedly made with only ordinary care.

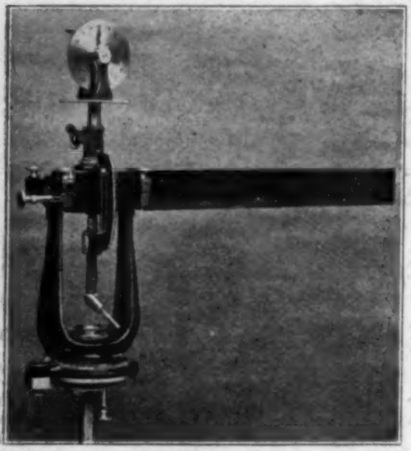
DATA.

	AMPERES.	VOLTS.	RESISTANCE BY CALCULATION OHMS.
Cell A	0.155	1.95	
Cell B	0.160	1.93	
Average	0.1575	1.94	12.32
Series	0.155	3.80	24.52
Parallel	0.305	1.94	6.36

THE ROTOSTAT AND GONIOSTAT: A TEACHERS' CLASS ROOM DEVICE FOR INSTRUCTION IN PROJECTION.

By Herman Hanstein,
Lane Technical High School, Chicago.

One of the most difficult tasks in the instruction of Projection is to demonstrate to the student how the mathematical outlines are obtained and drawn on the various planes of projection.



Rotostat Mounted by the Goniostat.

For more than fifty years teachers have used boxes with transparent (glass) sides in which the object to be represented was placed. The foot points of ordinates drawn from every point of these objects to the sides of the box and there connected by lines produced the mathematical outlines of the object—the top view, or plan, the front view, or elevation, and the side views, or side elevations.

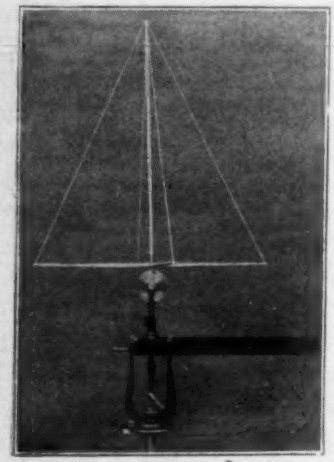


Fig. 1.

Another arrangement to accomplish the same purpose consisted in the substitution of fine wire screens for the glass sides of the box, which permitted the teacher's chalk marks to demonstrate the theories of ordinates from each point of the object to the vertical and horizontal sides of the surrounding box. These equipments will permit only a limited number of students to follow the demonstrations at a time and in large classes it is a serious waste in the modest allowance of time assigned for the instruction of projection on the programs of most of our schools.

With these arrangements progress in large classes is very slow and the need of other equipments with more extended facilities and possibilities is becoming apparent.

The search for efficient means to meet the demands of large classes resulted in the construction of a complete set of models in skeleton form, each two feet high, light and convenient to handle, with which the regular and an endless number of irregular surfaces, solids and their penetrations may be mounted before the class.

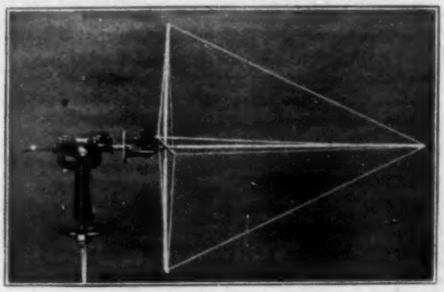


Fig. 2.

One of these models, the square pyramid, is shown in cut, (Fig. 1) inserted with a taper shank in the socket of the goniostat, an instrument having a vertical and a horizontal axis with divided circles to place the model in any desired position in space. The goniostat is mounted on the horizontal square axis of the rotostat, which permits the rotations of the model and goniostat in vertical and horizontal planes, with spring stops at quarter revolutions. Fig. 1 shows the model in its height and widths, the elevation or "front view."

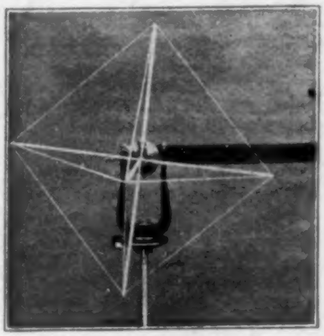


Fig. 3.

Fig. 2 shows the model rotated in vertical circles forward 90°, and represents its width and depth, the plan or "top view."

The revolutions of the model in horizontal circles are accomplished by the vertical axis of the rotostat, which in Fig. 3 shows side views, or side elevations.

It is the author's intention to give our pupils, with the aid of this device, a substantial and reliable foundation of this most important branch of a technical education—technical drawing.

The practical demonstrations of the models, their analyzation, their sections and penetrations, and their mathematical representations by drawing, soon improves the mental ability of the

student to "imagine" the existence of an object of any form in any position in space, which ability must be cultivated in the course of the education under discussion, as it forms so important an item in solving the problems that fall within its scope.

This equipment is in successful service in Room 313, one of the freshman drawing rooms of the A. G. Lane Technical High School, and colleagues who are interested in class room devices for the demonstration in geometry and projection are cordially invited to witness its operations during instruction hours and observe the educational results.

PACIFIC COAST ASSOCIATION OF CHEMISTRY AND PHYSICS TEACHERS.

The semiannual meeting of the association was held at the University of California, Berkeley, on Saturday, July 10, 1909. The attendance at both sessions was excellent and much interest was shown in the discussions. President A. G. Van Gorder delivered an opening address on the organization of science in the high school. Mr. S. E. Coleman of the Oakland High School discussed the question: "What readjustment, if any, is desirable in the physics course of the high school, in view of the changes in university requirements?" General discussion followed. Much interest was manifested in the topic opened by Dr. Winthrop Osterhout of the University of California. This involved the recommendation of a general elementary science course in the first year of the high school and included suggestions as to the purpose and character of such a course. Mr. N. L. Gardner of the Los Angeles Polytechnic High School, gave an account of the general science course as conducted during the past two years under his guidance. In the numerous opinions presented in general discussion, one idea was frequently expressed: the need for more humanizing of science work in public schools.

The association received an urgent invitation to unite with the American Federation of Teachers of the Mathematical and Natural Sciences. This was accepted without dissenting vote. Dr. William Conger Morgan of the department of chemistry, University of California, was elected to represent the association on the editorial staff of School Science and Mathematics. In the regular election of officers for the ensuing year, Mr. A. G. Van Gorder was elected president, and Mr. G. C. Barton secretary and treasurer.

At the conclusion of the formal sessions, a very pleasant social time was made pleasanter by a liberal distribution of refreshments. Quite a number of new names were added to the membership roll.

G. C. Barton, Secretary.

EARLY FORMS OF A FEW COMMON INSTRUMENTS.

BY WILLIAM E. STARK,

Ethical Culture School, New York.

In the books on practical geometry of the sixteenth and seventeenth centuries, one finds descriptions of many ingenious instruments. Some have been superseded by more exact and more complicated devices, others became obsolete as skill in numerical computation increased, but a few remain unchanged in principle—the same instruments in spite of change of form for convenience or accuracy. Through the courtesy of Professor David Eugene Smith of Columbia University, whose collection of old mathematical books and instruments is a wonderful field for the historical student, I am able to reproduce cuts of three of these survivors in the march of progress.

In "Seconda Squara Mobile et Arithmetica," written by Antonio San Giovanni and printed in Vicenza in 1686, the author describes as his own invention a pair of *parallel rulers* which is identical in form with the instrument still in use. Figure I is a

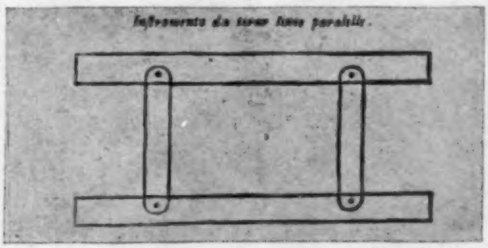


Figure 1.

FROM *Seconda Squara Mobile* BY ANTONIO SAN GIOVANNI, VICENZA, 1686.

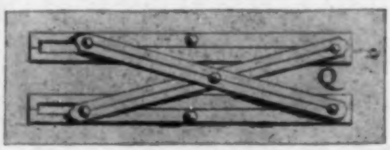


Figure 2.

PARALLEL RULERS. FROM *Traité de la Construction et des Principaux Usages des Instruments de Mathematique* BY N. BION.
SECOND EDITION. THE HAGUE, 1723.

reproduction of the cut given in San Giovanni's book. Figure II shows a second form of the same tool taken from a treatise on mathematical instruments published in 1723.

The *pantograph* is said to have been invented by P. Christoforo Scheiner,[1] who called it a "parallelogram." Scheiner and others who described the instrument evidently expected for it a wider application than it permanently held, for they show methods of adapting it to sketching landscapes and reproducing pictures as well as to enlarging and reducing maps and plans. Figures 3 and 4 showing the construction and use of the parallelogram, are taken from a book of the middle of the seventeenth century.

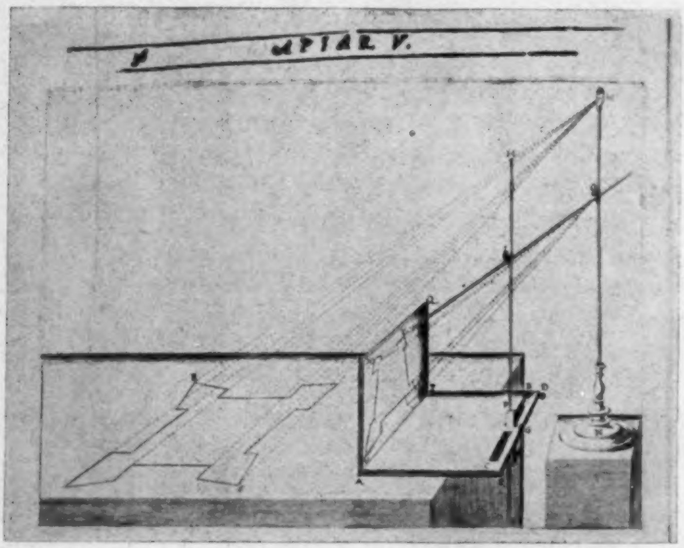


Figure 3.

FROM *Apiaria Universae Philosophiae Mathematicae* BY M. BETTINUS, BOLOGNA, 1645.

In "Neuerfundenes Instrumentum Universale" by Johann Matthes Bilern printed in Jena in 1696, the author describes a semicircular slide rule, which, if accurately constructed, would be nearly as efficient for some processes as the instrument in common use to-day.

Referring to the illustration (Figure 5) the semicircle ABCD

[1]Del Catasto Romano e di Alcuni Strumenti Antichi di Geodesia. E. N. Legnazzi, 1885

Figure 4.

FROM BETTINUS. (SEE NOTE UNDER FIGURE 3.)

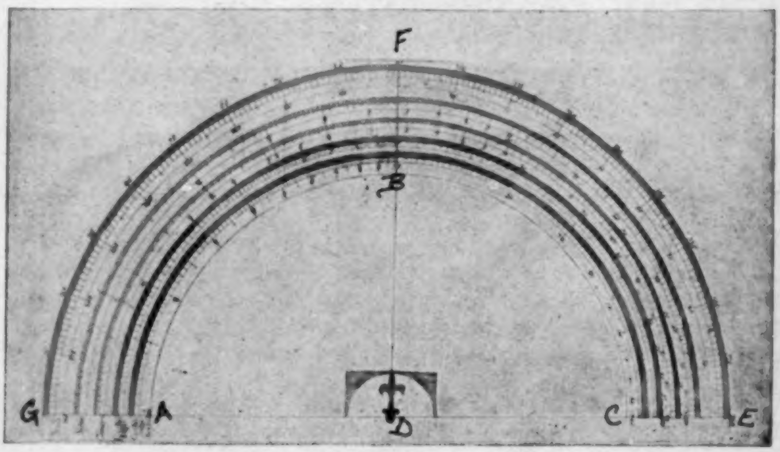


Figure 5.

SLIDE RULE, 1696.

may be turned on its center D within the semicircular band ABCEFG. On the outer edge of the semicircle and the inner edge of the band are corresponding logarithmic scales exactly like those of modern slide rules. The outer edge of the band is marked with a scale of degrees and within this are logarithmic scales of sines and tangents corresponding to the logarithmic scales of numbers mentioned above.

Anyone who understands the principle of the slide rule needs no explanation of the method of multiplying or dividing numbers with this instrument. For multiplying a number by a sine or tangent, a thread, attached to the center of the instrument, was stretched across the proper point on the sine or tangent scale and the semicircle was turned until the zero of the scale of numbers fell under the thread. The reading was then taken as in computation with ordinary numbers. For taking square or cube roots the thread was stretched across the given number, as it appeared on the outer scale of numbers. The reading on the scale of degrees was then divided by two or three and the thread stretched across the resulting number on the scale of degrees. The thread then indicated the result on the scale of numbers.

BIBLIOGRAPHY AND INDEX TO NORTH AMERICAN GEOLOGY FOR 1906 AND 1907.

A Comprehensive Catalogue.

The literature relating to the geology of North America comprises an immense mass of material and numbers so many contributions that the task of keeping it properly indexed has become formidable. The Geological Survey has already published indexes covering the years 1732-1905, and has just issued an index for the years 1906 and 1907. This index is published as Bulletin 372, and was prepared by F. B. Weeks and J. M. Nickles.

The articles indexed have appeared principally as books, magazine articles, and government and state reports. Many of the papers were published abroad, however, some of them in foreign languages, for the interest in American geology is world-wide.

The bulletin consists of 317 pages and the matter is classified under three principal heads—bibliography, index, and lists. The lists include references to chemical analyses and to descriptions of geologic formations, minerals, and rocks.

This bulletin, which is indispensable to all students of American geology, may be obtained free of charge by applying to the Director of the Survey at Washington, D. C.—*U. S. Geol. Survey.*

A CHAPTER IN ELEMENTARY TRIGONOMETRY.

By W. H. Jackson,
Haverford College, Haverford, Pa.
Introduction by Professor Wm. F. Osgood, Harvard University.

The following proof of the addition theorem in trigonometry, with the theorems leading up to it, which Professor Jackson has thrown into clear and simple form, is in substance that which was given in 1849 by De Morgan,* a mathematician of clear insight and high appreciation of scientific methods in mathematics. The proof has distinct advantages over those now current, since it is based on a principle that is fundamental in analytic geometry (Theorem I, below) and since, morever, it is general, including all cases. The practical advantage of this latter point is important. The methods of trigonometry and analytic geometry are general, and the student should come early to the conviction of this truth. It is not always desirable to work through all the cases that must be considered to make a proof complete. But when, as in the present case, a proof can be so arranged as to be at once simple and complete, it is worth while for teachers to avail themselves of this advantage. There will always be cases enough left in which the piecemeal plan is the only one simple enough to be used.

William F. Osgood.

1. Segments.

The position of any point on a given straight line is determined if we know, (1) the distance which separates it from a given point on that line, (2) the sense in which this distance is to be measured (e. g. up or down, to the right or to the left). The distance may be indicated by a positive number and the sense by a sign. The position of any point B relative to another point A is therefore determined by a number which we call the *algebraic length of the segment* AB, and which we shall denote by the symbol $\overline{AB}$. The bar is a reminder that the letters denote a number, not a line.

Note that the symbol $\overline{AB}$ has no meaning, until we have fixed (1) the unit distance and (2) the positive sense, and that the sign of $\overline{AB}$ is reversed when the positive sense is reversed. By

*Trigonometry and Double Algebra. London, 1849.

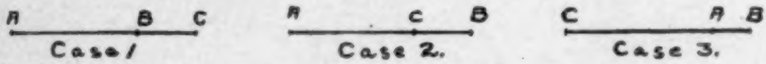
"the distance from A to B" we mean $\overline{AB}$, which is not the same thing as "the distance from B to A" or $\overline{BA}$, for $\overline{AB} = -\overline{BA}$.

The relations implied by the above notation are all summed up in the following theorem.

Theorem I. $$\overline{AB}+\overline{BC}=\overline{AC}, \qquad (1)$$
where A, B, C are any points in a straight line.

The student should work through many numerical examples to convince himself of the generality of this statement. To make sure that all possible cases have been considered, note that, given AB, three cases arise as C lies within the segment AB or to one side or the other outside it.

First suppose that AB is positive. The three cases become



(1) BC, AC positive, AC is numerically the sum of AB, BC;
(2) BC negative, AC positive, AC is numerically the difference of AB, CB;
(3) BC, AC negative, AC is numerically the difference of CB, AB.

If the positive sense be reversed, the numerical relation between the segments remains unaltered; for an algebraic identity is unaffected by changing the sign of every term.

All together there are six cases, the number of permutations of three things taken two at a time.

Beside the segments above defined we wish also to include nil segments:

$$\overline{AA}=0.$$

The theorem is readily seen to hold in this more extended case, and to admit, moreover, the symmetric form:

$$\overline{AB}+\overline{BC}+\overline{CA}=0.$$

2. Angles.

The position of any straight line which passes through a given point is determined if we know (1) the angle which separates it from a given straight line through that point, (2) the sense in which this angle is to be measured (e. g. clockwise or anti-clockwise). The angle may be indicated by a positive number and the sense by a sign.

The position of any straight line OB relative to another

straight line OA is therefore determined by a single algebraic number. This number we call the *algebraic magnitude of the angle* AOB, or more simply, the *angle* AOB, and we use the symbol $\overline{AOB}$ to denote this number. The bar is a reminder that the letters denote a number as before. Note that the symbol $\overline{AOB}$ has no meaning, until we have fixed (1) the unit angle, (2) the positive sense. For convenience, unless otherwise stated, we shall suppose that the unit is a degree. As before, $\overline{AOB}$ changes sign when the positive sense is reversed. By "angle which OB makes with OA" we mean $\overline{AOB}$, whilst "angle which OA makes with OB" means $\overline{BOA}$ and $\overline{BOA} = -\overline{AOB}$. Again the implied relations are summed up in a single theorem.

Theorem II. $$\overline{AOB}+\overline{BOC}=\overline{AOC}, \qquad (2)$$
where A, B, C, O are any points in a plane, O being distinct from A, B, C.

We introduce nil angles and thus have as before

$$\overline{AOB}+\overline{BOC}+\overline{COA}=0.$$

The underlying principle in each of these theorems is the same, but there is a difference as regards the correspondence between number and segment in one case and that between number and angle in the other.

Whereas any segment can be indicated by only one number, when a positive unit segment has once been given, any angle can be indicated by an unlimited series of numbers such that the difference between any two is a multiple of a complete revolution. As in what follows we are only concerned with the *positions* of lines and these are unaffected by a complete revolution, this difference is of little account. All we need note is that given the numerical values of $\overline{AOB}$, $\overline{BOC}$ there is only one value of $\overline{AOC}$ which will satisfy equation (2).

It is important to note that to every theorem about a segment AB in the first section there corresponds one about an angle AOB in the present section.

3. Projection.

Definition 1. The projection of a point P on a straight line is the foot of the perpendicular drawn from P to that straight line.

Definition 2. The projection of a straight line PQ on any other straight line is $\overline{P'Q'}$, where P′, Q′ are the projections of P, Q respectively.

Theorem III. If P, Q, R be any three points not necessarily in one straight line,

$$\text{proj. PQ}+\text{proj. QR}=\text{proj. PR}. \qquad (3)$$

For this is the same thing as

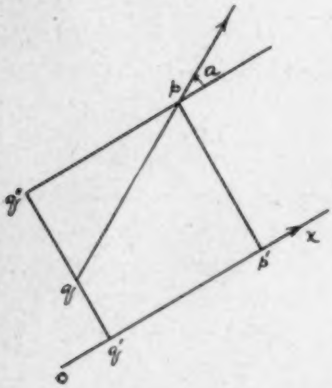
$$\overline{P'Q'}+\overline{Q'R'}=\overline{P'R'},$$

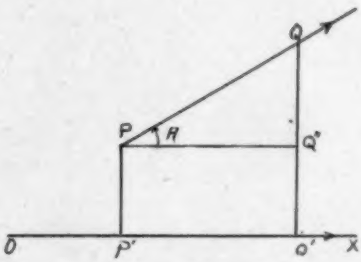
and this is true from Theorem I.

Theorem IV. If PQ is any segment, the ratio

$$\frac{\text{proj. PQ on OX}}{\text{PQ}}$$

depends only on the angle A which the positive sense in PQ makes with that in OX.





Let P′, Q′ be the projections on OX of P,Q respectively, and let a straight line through P, parallel to OX, cut QQ′ in Q″. Let small letters denote any similar figure in which

$$a=A.$$

It is required to prove that

$$\frac{\overline{P'Q'}}{\overline{PQ}}=\frac{\overline{p'q'}}{\overline{pq}}.$$

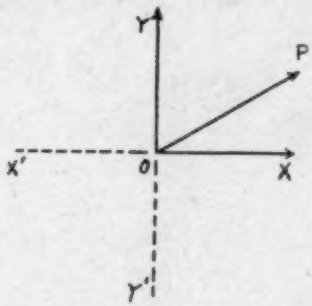
Without regard to sign, these ratios are equal because they are equivalent to PQ″/PQ, pq″/pq respectively. These latter ratios are equal because the right angled triangles PQQ″, pqq″ are equiangular and therefore similar.

So far as sign is concerned, $\overline{P'Q'}/\overline{PQ}$ is positive or negative according as the angle A when reduced, if necessary, by

multiples of 360° so as to lie between 180° and —180°, is numerically less than or greater than a right angle. The angles A, *a* are equal and therefore the two ratios have the same sign.

The last theorem states that a certain ratio has the same value for all angles of the same magnitude. For positive angles less than a right angle this ratio is supposed to have been already defined as cos A. Previous definitions can now be generalized as follows:

Definition 3. Let OX be called the initial line, and let XOY be a positive right angle. XOY is called the *frame of reference.*



Let any line OP make an angle A with OX and let PQ be any segment on OP or OP produced. Then

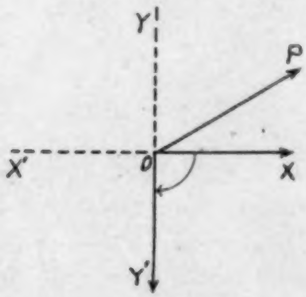
$$\cos A = \frac{\text{proj. } \overline{PQ} \text{ on } OX}{\overline{PQ}}, \quad \sin A = \frac{\text{proj. } \overline{PQ} \text{ on } OY}{\overline{PQ}},$$

$$\overline{PQ}\cos A = \text{proj. } PQ \text{ on } OX, \quad \overline{PQ}\sin A = \text{proj. } PQ \text{ on } OY. \qquad (4)$$

It is understood that $\overline{OX}$, $\overline{OY}$, $\overline{OP}$ are all positive.

4. Frames of Reference.

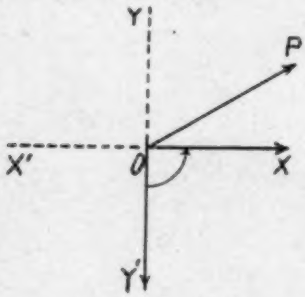
Let it be understood that X', Y' are respectively points on OX, OY produced backwards, so that $\overline{OX'}$, $\overline{OY'}$ are negative.



(1) Suppose the frame of reference turned over on the axis OX. The initial line is still OX but the positive right angle is now XOY′. If in the first case $\overline{XOP}$ is denoted by A, in the second case it must be denoted by —A.

Thus

$$\cos(-A)=(\text{proj. } PQ \text{ on } OX)/\overline{PQ}, \quad OX \text{ positive,}$$
$$=\cos A, \text{ from (4),} \qquad (5)$$
$$\sin(-A)=(\text{proj. } PQ \text{ on } OY')/\overline{PQ}, \quad OY' \text{ positive,}$$
$$=-\sin A, \text{ from (4).} \qquad (6)$$



(2) Suppose the frame of reference rotated backwards through a right angle. The initial line is now OY′ and the positive right angle is now Y′OX. If in the first case $\overline{XOP}$ is denoted by A, now $\overline{Y'OP}$ must be denoted by 90°+A. For, from (2),

$$\overline{Y'OP}=\overline{Y'OX}+\overline{XOP}=90^\circ+A.$$

Here

$$\cos(90^\circ+A)=(\text{proj. } PQ \text{ on } OY')/\overline{PQ}, \quad OY' \text{ positive,}$$
$$=-\sin A, \text{ from (4),} \qquad (7)$$
$$\sin(90^\circ+A)=(\text{proj. } PQ \text{ on } OX)/\overline{PQ}, \quad OX \text{ positive,}$$
$$=\cos A, \text{ from (4).} \qquad (8)$$

From these last four equations can be deduced at once the formulas for the sine and cosine of any angle obtained from A or —A by adding or subtracting any multiple of a right angle.

In (7), (8) write 90°+A=B, A=B—90°.

$$\cos B=-\sin (B-90^\circ). \qquad (9)$$
$$\sin B=\cos (B-90^\circ). \qquad (10)$$

In (7), (8) write —A for A, and apply (5), (6).

$$\cos (90^\circ-A)=-\sin (-A)=\sin A. \qquad (11)$$
$$\sin (90^\circ-A)=\cos (-A)=\cos A. \qquad (12)$$

In (7), (8) write A=90°+B, and again apply (7), (8).

$$\cos(180°+B) = -\sin(90°+B) = -\cos B. \quad (13)$$
$$\sin(180°+B) = \cos(90°+B) = -\sin B. \quad (14)$$

In (13), (14) write $-B$ for B, and apply (5), (6).

$$\cos(180°-B) = -\cos(-B) = -\cos B. \quad (15)$$
$$\sin(180°-B) = -\sin(-B) = \sin B. \quad (16)$$

The writer believes that it is best for the student to obtain these results for himself direct from the figure, as he requires them, rather than to memorize a large number of results. The method of direct proof for two of these results is indicated below.

Suppose the frame of reference is turned over on the axis OY. The initial line is now OX′ and the positive right angle is now X′OY. If in the first case $\overline{XOP}$ is denoted by A, now $\overline{XOP}$ must be denoted by $-A$. Further, we now have from (2)

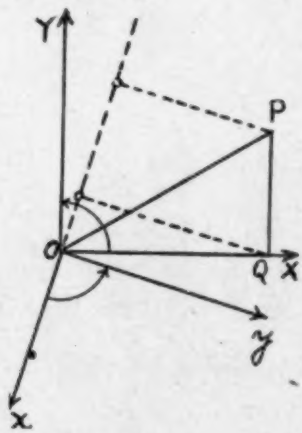
$$\overline{X'OP} = \overline{X'OX} + \overline{XOP} = 180° - A.$$

Suppose, secondly, that the frame of reference is turned over on the bisector of the angle XOY, so that the positions of the axes are interchanged. The initial line is now OY and YOX is now a positive right angle. Again if $\overline{XOP}$ in the first case be denoted by A, in the second case it must be denoted by $-A$. We have, further, in this case, from (2)

$$\overline{YOP} = \overline{YOX} + \overline{XOP} = 90° - A.$$

5. The Addition Formulas.

In accordance with the method just developed, we can find the effect of adding an angle B to an angle A by turning the frame of reference through an angle $-B$. Let the new position of the frame be denoted by xOy.



Let $\overline{XOP} = A$ and let $\overline{xoX} = B$. From equation (2),

$$\overline{xOP}=\overline{xOX}+\overline{XOP}=A+B, \qquad \overline{xOY}=90^\circ+B.$$

Let Q be the projection of P on OX and let the positive direction in QP make the same angle with OX which OY does. The required formula is obtained by applying equations (3) and (4). The former gives, with reference to any straight line

$$\text{proj. } OP=\text{proj. } OQ+\text{proj. } QP.$$

And now we obtain from (4), firstly, with reference to Ox,

$$\overline{OP}.\cos(A+B)=\overline{OQ}.\cos B+\overline{QP}.\cos(90^\circ+B).$$

That is, $\cos(A+B)=\dfrac{\overline{OQ}}{\overline{OP}}\cos B+\dfrac{\overline{QP}}{\overline{OP}}\cos(90^\circ+B)$

But from (4),

$$\overline{OQ}/\overline{OP}=\cos A, \qquad \overline{QP}/\overline{OP}=\sin A,$$

and from (7),

$$\cos(90^\circ+B)=-\sin B.$$

Therefore,

$$\cos(A+B)=\cos A\cos B-\sin A\sin B. \tag{17}$$

Secondly, with reference to Oy,

$$\overline{OP}.\sin(A+B)=\overline{OQ}.\sin B+\overline{QP}.\sin(90^\circ+B).$$

But from (8),

$$\sin(90^\circ+B)=\cos B.$$

Therefore,

$$\sin(A+B)=\cos A\sin B+\sin A\cos B. \tag{18}$$

The proofs of all these formulas have been perfectly general so there is nothing to hinder us from changing B into —B in equations (17) and (18) and applying (5) and (6) to give the formulas for $\cos(A-B)$, $\sin(A-B)$. Again by making B a multiple of a right angle the formulas (9) to (16) may be proved.

In particular, putting $B=-A$ in (17) we obtain

$$1=\cos^2 A+\sin^2 A. \tag{19}$$

Note also that if in (17) we replace B by $B-90^\circ$ we obtain (18) when we apply (9) and (10).

LOCI PROBLEMS IN GEOMETRY.

From the Point of View of a Few Secondary Schools.[1]

BY MR. ERNEST G. HAPGOOD,
Girls' Latin School, Boston.

The proper place of loci problems in geometry has been attracting the attention of the teachers of mathematics in the secondary school in recent years with increasing insistence. Formerly treated as a topic of interest, not of vital importance to the structure of geometry, but incidentally because of its value in illustrating the generalized form of statement and its usefulness as a tool for the solution of construction problems, the locus problem has of late assumed front rank, at least in the minds of those college authorities who set the entrance papers in mathematics, and has come to demand a large share of the time and energy devoted to the study of geometry.

It is not my purpose to discuss methods of teaching or of solving loci problems but to point out briefly the attitude of a few secondary schools toward loci problems from the standpoint of college entrance requirements. I lay this emphasis upon the entrance examination because this question is not one that had its origin in the secondary schools but has been forced upon preparatory schools by the colleges whose entrance papers have called for proficiency in the solution of difficult loci problems. Nor have the teachers of the subject been given any assistance in the way of definite statements of what the requirements in loci are. They have been left to infer, from the nature of the questions that appear on the examination papers, the character and scope of the requirements in loci. I shall confine myself largely, therefore, to a discussion of the question as to whether or not this increased emphasis on loci problems is justified. The following points would seem to indicate that the emphasis is undue.

I. The inherent difficulty of the subject for the "average" pupil makes it unsuitable. All teachers have experienced the necessity of expending much time in impressing the concept of a locus upon the pupil's mind. The idea seems to the pupil vague and unnecessarily abstruse and he naturally shrinks from grappling with it. With all possible effort in the limited time available for this topic it is almost certain that over fifty per cent

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[1]Paper read at the sixth annual meeting of the Association of Mathematical Teachers in New England, December 5, 1908.

of the class will fail to acquire the ability to solve independently a locus problem of even ordinary difficulty while with the problems of an unusual degree of difficulty this percentage of failures is greatly increased and the majority of the class will feel it hopeless to make any attempt to solve the problem. So large a number of failures in any subject would naturally cause a teacher careful thought and most teachers of geometry feel that any great amount of attention to loci problems on the part of a whole class is time poorly spent.

II. The range of topics suitable for loci problems in elementary geometry is narrow and the "content" of the subject of loci is extremely limited. Partly as a consequence of this there is a tendency among examiners to overstep the natural limits of loci in geometry and to introduce problems belonging more properly to higher mathematics. This, we feel, is open to the same objection as the discarded custom of introducing into the subject of arithmetic questions more readily solved by the methods of algebra.

In seeking to determine the actual content of the subject of loci, I have examined many standard text-books and particularly the more recent publications of the American publishers of school books. The investigation was confined to the plane geometry in each case, since very little protest seems to have arisen against the trend toward loci in solid geometry. The results are as follows:

TEXT-BOOK	NO. OF LOCI PROBLEMS
Wentworth's Plane (Revised)	31
Beman & Smith	19
Wells' Essentials of Plane Geometry	8
Wells' New Plane Geometry	10
Robbins' Plane Geometry	23
Schultze & Sevenoak	11
Chauvenet (Byerly's Revision)	26
Shutt's Plane	15
Durell's Plane	24
Wentworth and Hill's Exercise Manual	29
Wentworth and Hill's Examination Manual	33

In addition to these sources there are two pamphlets on the subject of loci, published either for private use or by an individual school; one, containing 50 problems, by Mr. Nolan, the well-known Harvard tutor, the other, containing 79 problems, by the Newton High School Mathematical Department.

III. The nature of the requirements as has been suggested

already is obscure and ill-defined. The term locus, itself, is subject to looseness of definition and the varying conceptions of it are not always harmonious. The following definitions will illustrate this tendency:

1. All points in a plane that satisfy a single geometric condition lie, in general, in a line or group of lines; and this line, or group of lines, is called the locus of the points that satisfy the given conditions. Wentworth, p. 44.

2. A line is the locus of all points that satisfy a given condition, if all points in that line satisfy the given condition and no points out of that line satisfy it. Phillips and Fisher, p. 55.

3. The place of all points satisfying a given condition is called the locus of points satisfying that condition. Beman & Smith, p. 80.

4. A geometric locus is the assemblage of all the points which possess a common property (*i. e.* satisfy the same geometric condition). Chauvenet, p. 22.

5. The locus of a point is the series of positions the point must occupy in order that it may satisfy a given condition. Robbins, p. 60.

6. The locus of a point is the path of a point moving according to a given geometric law. Durell, p. 52.

7. When the position of a point in a plane is limited to and may be anywhere in a line or group of lines, the line, or group of lines, is the locus of the point. Shutts, p. 65.

8. A locus of a point in a plane is a line or group of lines all points in which fulfill a *certain condition, fulfilled by no other points.*

Not only is the lack of a *standard definition* of the term "locus" apparent but a study of the questions set in examination papers reveals the lack of a clearly established standard as to the kind of problems to be offered for examination and the kind of solution that is expected. One of the best known of the secondary school teachers of geometry in greater Boston tells the story of a college professor whose duty it was to prepare entrance examinations in geometry, who informed him that he frequently lay awake nights in order to devise problems (particularly loci) that were *difficult* enough to warrant their use on the examination papers. This professor was proceeding, apparently, on the assumption that the same pupils were taking the examinations year after year and that the questions set should be of progressive difficulty.

It is not clear what kind of a solution of the locus problem is expected; whether a correct answer with an accurate drawing of the locus is sufficient, or whether some proof is expected and if so, whether the proof for one branch is adequate or the negative or converse should also be established. Many of the loci problems appearing on the recent examination papers have

been involved and difficult both as to solution and to proof of the accuracy of the locus determined. Moreover this class of problem is often independent of subsequent applications and therefore open to the same objection as much discarded algebra in that it is difficult in itself and has little or no practical bearing on the later developments of the subject as a whole.

The following statements have been furnished by representative teachers of mathematics in Boston and the suburbs who were asked to state their opinion as to the amount of emphasis to be laid upon loci problems in geometry, to furnish problems of the kind they considered suitable (and also of the kind considered unsuitable) and to suggest the kind of solution they thought desirable.

"If the average pupil in a secondary school were asked to divide locus problems into two groups, those which he is glad to meet, and those which he is not, his first group would include *all* locus problems, and the second would include the rest.

"The teacher, however, can make a different division. The first group includes problems where the form of the locus is readily determined by plotting points; thus the student's mind is directed first to finding what the locus is, and afterwards to testing the correctness of his answer by looking for the reasons. This group is suited to students of the secondary school age.

"The second group includes problems where the form of the locus is not readily determined by points; thus the student is forced to begin by searching for the reasons; and the locus appears *in toto* at the end of the investigation. This group is likely to be unsuited in examinations when the student is under the pressure of nervousness and limited time; and it is of doubtful value even in the class room: it means doing by elementary geometry what would better be left for analytics, and it is open to the same objection as the discarded practice of having problems done by arithmetic which belong to algebra.

"Between these two groups lies a group of problems which are suitable provided that they are introduced, i. e. directly introduced, by propositions which will act as hints.

"The following are examples of the three groups:

"*Suitable*. Given a square with the side 3 inches long. Find the locus of a point P such that the distance from P to the nearest point of the perimeter of the square is one inch. Describe the locus accurately.

"Given a fixed point D within a triangle ABC, Choose any

point, E, on the perimeter of the triangle, draw DE, and let P be the middle point of DE. Find the locus of P when E traces out the whole perimeter of the triangle. Describe the position of the locus exactly, and prove the correctness of your answer.

"*Suitable if introduced properly.* Through a point A on the circumference of a circle, chords are drawn. On each one of these chords a point is taken one third of the distance from A to the other end of the chord. Find the locus of these points, and prove that your answer is correct.

"A chord BC of a given circle is drawn, and a point A moves on the longer arc BC. Draw the triangle ABC, and find the locus of the center of a circle inscribed in this triangle.

"*Unsuitable.* A set of circles are tangent to a given line at a given point; and a second set of circles are tangent to a second line at the same point. What is the locus of the points of intersection of equal circles of the two families? Prove that your answer is correct.

"A parallelogram ABCD has the vertex A fixed and the directions of the adjacent sides AB and AD also fixed. Find the locus of the vertex C if the sum of the two sides AB and AD is constant, and prove your answer correct."

"The average candidate for entrance to college may reasonably be expected to know perhaps thirty elementary loci. He should be tested on rather simple applications of them, chosen with his necessary limitations clearly in view. Examiners should make plain the nature of answers expected by them; whether diagram, clear description, one branch of proof, complete proof, or some combination of these possibilities.

"The following questions seem sufficiently difficult:

"1. Find in a given straight line a point such that the tangents from it to a given circle shall have a known length.

"2. Determine the points, equidistant from two given intersecting lines, from which a given circle is seen under a known angle."

"To me a college examination represents a method used by the college to determine a candidate's fitness to enter the institution and benefit by the courses offered. Based on that understanding of it, some of the problems put on geometry examinations seem not quite defensible. For instance in June, 1907, the Massachusetts Institute of Technology puts on a paper two locus problems; to be sure one is a statement of the locus requiring proof, and the other asks for a locus, with the proof

evidently not required. However, those two questions are based on the locus, and one locus problem on a limited paper would serve all examination purposes.

"Further, it seems to me that the college examiners fail to get the very thing they are after, namely, knowledge of the pupil's power, when in a locus problem, so much time and ingenuity are required to find what the locus itself is that the average examinee will never have a chance to show whether he knows how to prove the truth of a locus statement or not. A locus problem is a very important kind of geometry problem, but an example on a college paper to be done in a limited time ought not to be so difficult as one where the student has his leisure to get it. It seems to me that the emphasis on any test is to be put on proving the truth of the statement of the locus, and that presupposes that the locus can be described."

"How would the following do?

"(a) If in a right triangle the acute angle is 30°, the hypotenuse is twice the short side. Prove.

"(b) What is the locus of points one inch from the line indicated on the diagram? Compute the length of the locus. (Diagram: An angle of 60° with arms 2 in. and 4 in. long.) This problem requires the study in determining the locus itself; not in proving that the solution is correct. That is where I believe the study should be required.

"Regarding the proof, the second part (that points off the line do not satisfy the condition) seems of little practical value; for one is usually satisfied if he has found the location of points which do satisfy, and the proof is usually very involved."

"What is the locus of the middle point of a line 5 inches long having its extremities in the opposite sides of a rectangle 8 inches long by 3 inches wide?

"In thinking over the possibilities of the subject of locus I find that the most satisfactory test of the pupil's knowledge of the subject seems to me to be a problem based on fundamental principles previously studied and not too far removed from them; and the pupil should be required to show that every point of the locus satisfies the conditions of the problem. As an example of this kind of problem, I would choose something like this from the entrance examination in geometry given by Harvard University in September, 1900.

"Through a point A in the circumference of a circle chords are drawn. On each one of these chords a point is taken one

third of the distance from A to the other end of the chord. Find the locus of these points and prove that your answer is correct."

"The solution of this problem should be easily deduced from the following which should be one of the fundamental problems of the subject. Find the locus of the middle points of all chords that can be drawn through a fixed point in a given circle. (Case when fixed point is in circumference.)

"There are good locus problems, too, which require a careful diagram and show relations of lines, demanding of the pupil a power of interpretation of statement similar to that required in the arrangement of the data for the solution of a problem in algebra. This kind, I think, should not require proof from the pupil.

"Here is one from the June, 1904, Harvard entrance examination: "Given a square with the side 3 inches long. Find the locus of a point P, such that the distance from P to the nearest point of the perimeter of the square is one inch. Describe the locus accurately." Here is a chance for two loci.

"But this kind can be and has been made much too difficult and in some cases proof has been required where the requirement of the answer alone made the question sufficiently difficult. Examples of this are found in the Harvard entrance examinations for June, 1903, and June, 1906.

"The subject of locus is an interesting and a fruitful one, I believe, and the principles can be introduced in many ways, in proof of theorems and in construction work, etc.; but it can be overworked just as the subject of graphs in algebra. It is very useful as a means of showing the pupil how a specific truth can be made general."

If, then, we are not to consider the locus problem in geometry as an end in itself but rather as a means to an end, the solution of the question as to what is the proper place of loci in elementary geometry lies in finding the principal end of the whole subject of which they form a part. Practically the only field in which loci are of great use is the solution of construction problems and therefore it would seem wholly fair from the standpoint of the secondary school, as well as adequate from the college viewpoint, to restrict those examination questions which have to do with loci to certain fundamental loci theorems and supplementary problems. The guiding principles in this selection should be simplicity, wide range of applicability, and serviceability to

the student as tools in other investigations. Construction problems might well be set which will test the ability of the pupil to apply the method of known loci to their solution. A simple illustration of problems of this character is, "To draw a circumference through a given point and tangent to a given line at a given point."

An excellent means of putting some such standard of loci problems into operation for examination purposes would be the adoption by the colleges of a list of loci theorems and supplementary problems, with which the candidate for admission might be expected to apply to the solution of construction problems; such a list might well be placed in a Syllabus of Required Propositions like that of Harvard or issued by the N. E. Board on College Entrance Requirements.

The following list of loci theorems and supplementary loci theorems for which I am indebted to the President of this Association is in use at the Newton High School and furnishes a good suggestion of what can be done to secure a uniform standard.

Loci Theorems.

1. The locus of a point equally distant from the extremities of a line is the perpendicular bisector of the line.
2. The locus of a point equally distant from the sides of an angle is the bisector of the angle.
3. The locus of a point at a given distance from a given point is the circumference having the given point as a center and the given distance as a radius.
4. The locus of a point at a given distance from a straight line is the line parallel to the line at the given distance from it.
5. The locus of the vertex of a right angle in a triangle with a given hypothenuse is the circumference of a circle having the given hypothenuse as a diameter.
6. The locus of the vertex of a triangle having a base of given length and a given vertical angle is the arc of a circular segment in which the given angle can be inscribed, and having the given base for a chord.
7. The locus of the middle point of a line drawn from a point terminating in a given line is the line parallel to the given line midway between the point and the given line.

Supplementary Loci Theorems.

1. Find the locus of the center of a circle which passes through two given points.
2. Find the locus of the center of a circle which has a given radius and passes through a given point.

3. Find the locus of the center of a circle which is tangent to a given line at a given point.
4. Find the locus of the center of a circle which is tangent to two intersecting lines.
5. Find the locus of the middle points of equal chords in a circle.
6. Find the locus of the middle points of parallel chords in a circle.
7. Find the locus of points which are the vertices of equal angles circumscribed about a circle.
8. Find the locus of the middle points of all chords in a given circle which pass through a given point.
9. Find the locus of the middle point of a line of given length which terminates in two perpendicular lines.
10. Find the locus of a point equally distant from two parallel lines.
11. Find the locus of the centers of circles which have a given radius and which are tangent to a given line.

Finally, in order that this discussion may lead to some definite action, looking toward the improvement of existing conditions, I will offer the suggestion that a committee be appointed by this association to investigate the question of the proper function and scope of loci problems in geometry, frame an adequate definition of the term, and draw up lists of fundamental and of supplementary loci problems that will furnish an adequate and clearly defined basis for work in geometrical loci and problems relating thereto.

LOCI PROBLEMS IN GEOMETRY.[1]

The Point of View of the College.

By Professor E. B. Wilson,

The Massachusetts Institute of Technology.

When I found that I was to speak to you to-day of the point of view of the college relative to problems on loci in elementary geometry, I was completely at a loss as to what to say; for I did not know that the college had any special point of view in regard to problems on loci. I did not know and I did not care. When, in this predicament, I turned to consult my colleagues in the department of mathematics at the Massachusetts Institute of Technology, it appeared that almost unanimously they, too, failed to recognize any special collegiate point of view and were

[1]Paper read at the sixth annual meeting of the Association of Mathematical Teachers in New England, December 5, 1908.

indifferent to its existence. That this fact should be established and emphasized seems highly worth while.

What we want for our candidates for admission to the Institute is a thorough, straightforward, useful training in geometry as a whole. Our chief difficulty is that the pupils who come to us cannot use their geometry, do not know mensuration, have meager geometrical intuitions; in short, do not see straight. In view of this condition it is distinctly unfortunate unduly to annoy the secondary school pupil with intricate, artificial, or freakish problems on loci or unduly to harass him with labyrinthine difficulties in limits, his other geometric (?) *bête noire*.

While speaking thus, I would especially disclaim representing all collegiate instructors and institutions. The fondness for fine points and the love of critical discussions are unquestionable reactions of a highly developed nervous system; and there are men and institutions that have their nervous systems so highly organized as to be on the point of disorganization, of prostration. The ordinary high school pupils, however, are not in this feverish condition, and it does not appear wholly desirable to urge them too insistently into it.

It is true that our candidates for admission have not a satisfactory knowledge of the idea of a locus, and the same is true of the idea of a limit; but I have found a similar state of affairs among my own and other students in the freshmen, sophomore, and even in the junior years. Some ideas develop slowly. It is necessary to keep constantly at them, but gently; for excessive pressure is as harmful to those ideas themselves as it is deleterious to the subjects which are crowded aside for their sake. The exaggerated emphasis and disproportionate worry which are often expended upon these topics may be due to the unfortunate fact that many secondary schools feel obliged to coach for our entrance examinations and, if possible, to anticipate them.

Entrance examinations are not worth talking about and I should prefer to say nothing about them; but that I might see what our papers at the Institute indicated with reference to loci, I took the pains to look over the examinations in plane and solid geometry for the last five and a half years. It appears that out of eleven papers in each subject, five in each contained a question on loci. As there are not less than six questions per

paper, you see that by our actions we estimate problems on loci at about eight per cent. This hardly appears excessive, perhaps hardly sufficient. On the whole, I am inclined to believe that this percentage is about what should be allotted to loci, and possibly as much to limits, in teaching plane and solid geometry.

Such estimates, however, are largely guess work and it will be better to return to the real question of interest: What shall we do about loci? I have two points to make: one relative to the definition of a locus, one relative to the construction of a locus.

The usual statement of the definition is this: A locus is a geometrical figure such that every point on it satisfies the conditions that determine the locus and such that no point not on the figure satisfies the conditions. This definition seems to me bad. We are not concerned with the points which are not on the figure or do not satisfy the conditions. I much prefer to state it thus: A locus is a geometrical figure such that every point on it satisfies the conditions and such that conversely every point which satisfies the conditions lies on it. The direct proposition and its converse seem preferable, in general easier to prove, and more obviously connected with the facts in which we are interested.

Take, for example, the proposition about the points equally distant from the ends of a segment. To show that every point in the perpendicular bisector is equally distant from the ends, requires the elementary theory of the isosceles triangle, and to show that every point which is equally distant from the ends is upon the bisector requires merely the same figure and the same theory; nothing could be simpler or more natural. As usually given, the complete proof introduces points which neither satisfy the conditions nor lie on the bisector, requires the use of theorems which have but a remote connection with isosceles triangles, and forces the pupil to operate with inequalities. This is all needless: it may be instructive, but it is useless and confusing. It is very seldom that we have to use inequalities, and it is well to avoid them here where there are other difficulties.

It may be noted that once we pass above the grade of high school instruction there is a strong tendency to let up on the enforcement of the double-ended proof relative to the locus; a great many good people are entirely satisfied with giving one half of the proof alone. We are, in fact, very much less concerned over the rigorous necessity for a two-sided proof than we are over ascertaining the geometric facts as to what the locus

is and how it is situated relatively to the rest of the figure. And this brings me to the second point, the necessity, the absolute necessity that the student should construct the locus, that is, should draw with reasonable accuracy enough points of the locus so that he may see clearly what the locus is and how it is situated.

The students who come to us exhibit an almost perfect inability to make and use a figure. Of the hundreds of entrance papers I have read, all except a pitiable few have borne clear evidences of this inability. From our point of view at the Institute, it is very important for the sake of our drawing that the condidates for admission should have some idea of the use of a ruler and compass, and for the sake of our instruction in analytical geometry it is essential that our entering class should have the habit of making a preliminary sketch of a locus or other figure as a qualitative and semiquantitative check on their analytic work.

So important do I regard work in construction, and in computation, for pupils in the secondary schools that I should be glad to see more questions on these matters on our entrance papers. In saying this I do not wish to be interpreted as outlining any actual intention on our part; I merely wish to state as strongly as I may to you what appears to me desirable in instruction.

Discussion Opened by Mr. William Fuller,

Mechanic Arts High School, Boston.

In the few minutes which I may appropriate, I ask your attention to some additional sources of information and material concerning loci. You will perceive that elsewhere, as with us, the subject of loci viewed from the standpoint of elementary plane and solid geometry, gets but scant treatment. For a number of years my practice has been to make special note of material on loci encountered in American, English, French, and German texts on geometry that have come in my way; but nowhere has adequate exposition and illustration yet been found. In December, 1906, I addressed inquiries to a number of mathematical professors in New England colleges asking, first, comment on the apparent increase of emphasis laid on the subject of loci in college entrance examinations, and second, for references to books either in English, French, or German giving satisfactory treatment of the subject. Of those replying some had not observed the alleged increase of loci questions in entrance papers; others had observed it, but thought it merely an expression of idiosyncrasy in the individual examiner.

Most had no books to recommend; a few sent helpful references, but all were agreed that no adequate treatment of the subject was known to them. One of my correspondents kindly but firmly insisted that, whatever I might mistakenly *think* I wanted to know, that which I really needed was to make myself familiar with the treatment of loci as developed in the current texts on Analytical Geometry. When by further writing I was happily enabled to show that his diagnosis of my case was erroneous, he concurred in the generally expressed opinion that no good treatment of the subject from the elementary point of view is available; and said that he knew no reason why colleges in their entrance examinations should lay increasing stress on the subject of loci. Permit me now, with all suitable dryness and brevity, to specify a few sources of information concerning loci:

1. Leçons de Geometrie Elémentaire par Jacques Hadamard, Paris, has about thirty (30) theorems and problems scattered through its 305 pages. Of these a considerable number are concerned with topics commonly absent from our American text-books. (Reciprocals, polars, harmonics.)

2. Ebene Geometrie von G. Mahler, Leipzig, (Sammlung Göschen Nr. 41) has some forty (40) loci theorems and problems. Of especial interest is the section called Method of Loci in Part III on the Nature and Treatment of Exercises.

3. The famous Hauptsätze der Elementar Mathematik, Berlin, by Dr. F. G. Mehler, long used as a basis of mathematical work in the German Gymnasia, gives slight attention to loci, containing altogether only about fifteen (15) theorems and problems of the most elementary character.

4. The Theorie der Geometrischen Konstruktionen, August Adler, Leipzig (Sammlung Schubert, Nr. 52), pp. 14-22, Methode der Geometrischen örter, gives eight (8) fundamental theorems on loci, and applies them to the solution of some nineteen problems. Dr. Adler shows the noble impatience of elementary tasks characteristic of the accomplished mathematician, proceeding into fields not tillable to advantage in our secondary schools.

5. In Methods and Theories for the Solution of Problems in Geometrical Constructions by Julius Petersen, eleven (11) fundamental loci are given which are applied to the solution of 116 problems. This book, originally written in Danish, has been translated into French, German, and English, London, 1879. (Sampson Low, Marston, Searle and Rivington.) It is an admirable book, small but rather expensive.

6. Elementary Geometry, Practical and Theoretical, Godfrey and Siddons, Cambridge University Press, offers about thirty (30) theorems on loci in Elementary Plane Geometry, twenty (20) on higher curves, and perhaps 80 or 90 exercises whose solutions involve loci. These have a wide variety in degree of difficulty, ranging from this exercise: "What is the locus of a man's hand as he works the handle of a common pump?" or "What is the locus of a door handle as the door opens?"; to this: "OA, AP are two rods pointed at A. OA revolves about a hinge at O, and AP revolves twice as fast as OA in the same direction. Find the path of a point on AP." You see at once that the loci in

question are forms of the limaçon, one being a cardioid, and you recall that the limaçon is a curve of the fourth degree invented by Pascal and having for its polar equation $r = a \cos \theta + b$. These more difficult loci are not treated theoretically but are plotted by means of coördinates and tracing paper.

7. Practical Plane and Solid Geometry, J. Harrison, London, (Macmillan). In this book the notion of a locus is briefly defined and explained, a few elementary and a few more difficult loci are given for plotting on determination by trial. Among these are the cycloid, the epicycloid, the hypocycloid, and the involute of a circle.

8. Practical Plane and Solid Geometry, Morris and Husband, London, Longmans, Green & Co. In this book a brief chapter is devoted to loci. Definitions, explanations, 22 problems, and five additional exercises are given. The problems deal in part with linkages, the simpler loci utilized in machinery, and include, besides, the Archimedean spiral, the ellipse, the hyperbola, and the parabola. Most of the material is unsuited for use in college entrance examinations.

9. Elements of Geometry, University Edition, Part III, Introduction to Modern Geometry, Edward Olney. This book now out of print, but occasionally to be picked up in second-hand book stores in Chicago or Ann Arbor, has 28 theorems and problems on loci. A valuable feature is the detailed study (pp. 293-4) of a moderately difficult locus problem, showing how a pupil might proceed to attack the solution.

It is interesting and important to note variations in the definition of a locus as proposed by these different writers:

Petersen gives no definition, Adler gives one by implication only. Mahler says (p. 11), "When the position of a point, by reason of the conditions which it must fulfill, is not exactly determined, it is nevertheless often restricted to a line in which the point must lie. This line is called the locus of the point." Mahler writes, (p. 15) "If a line represents the totality of all points which satisfy a given condition, it is called the locus of this point."

Hadamard defines as follows: (p. 2) "The locus of a point which may occupy an indefinite number of positions is the figure, in general a line or a surface, formed by the ensemble of its positions."

Godfrey and Siddons, (p. 144), say: "If a point moves so as to satisfy certain conditions, the *path* traced out by the point is called its *locus*."

In the Elements of Plane Geometry, (Association for the Improvement of Geometrical Study) this definition is found: (p. 81) "If any and every point on a line, part of a line, or groups of lines, satisfies an assigned condition, and no other point does so, then that line, part of a line, or group of lines, is called the *locus* of the point satisfying that condition."

Olney, (p. 288), says: "The locus of a point is the line generated by the motion of the point according to some given law. In the same manner a surface is conceived as the locus of a line moving in some determinate manner."

It is instructive to place alongside these definitions the more generalized formulations of analytic geometry. Briggs, in his Elements

of Analytical Geometry, (p. 14) says: "The locus of an equation is the path of a point which moves in the plane so that at each instant its coördinates satisfy that equation when substituted therein for x and y, the variables in that equation."

Harrison in his Treatise, (p. 33) writes: "A *Locus* is the series of positions, real or imaginary, to which a point is restricted by given conditions of form."

However inadequate the treatment of loci may be in our text-books in plane geometry (and it certainly is slight, cursory, and scattered) I can indeed recall at this moment but one book, Pettee's Plane Geometry, in which the matter is properly assembled for review or accessibility; its treatment in our books on solid geometry is characterized, when it has substance enough to have a character, by still greater incompleteness. Most of our writers with whom I am acquainted do not even take the trouble to extend their definition of a locus to meet space conditions, while they give very few and much scattered problems. The only German text I have recently examined with reference to loci is the admirable little treatise on Stereometrie by Dr. Glaser. This appears to contain no more than three loci problems and no definition or exposition; as compared with the 40 theorems and problems in Mahler's Ebene Geometrie of similar scope.

It appears to me indubitable that examiners should confine themselves to few and elementary principles with applications of quite moderate difficulty. If they translate material from French and German sources, as they no doubt sometimes do, they should not neglect to translate into the American point of view as well as into the English language. It is obvious that a French geometrician would be likely to state his problems from a viewpoint that might render them, in a literal translation, unintelligible to American youth.

It is the impression of a friend of mine that the college examiner apostrophizes himself thus: "Go to now! This youth must, by this time, be able to do a harder example than the one he failed on last year!" unmindful that this year it is a new boy no more capable or mature than last year's candidate.

I heartily concur in the suggestion of the essayist that this association, perhaps through the medium of its council, should consider the desirability of publishing a brochure on loci, and I hope that at an appropriate moment someone will feel moved to offer a resolution to that purport.

BY MISS MARY F. GOULD,

Roxbury High School.

Most text-books treat fairly simple loci problems as something new and unusual in the pupils' experience. I agree with Professor Wilson that the difficulty lies in the definition of the term locus. The trouble in solving loci problems arises primarily from the fact that pupils do not know exactly what they are trying to do. The class is given a definition followed by problems, which in many cases are exceptions

to the rule. For example some of the definitions this morning, if I recall them, do not include cases where the locus is a point, a group of points, or a surface.

In my own classes we consider these problems early in the year. Soon after defining a geometrical figure as any combination of points, lines, and surfaces, and developing the fact that a moving point generates a line, a moving line a surface, we decide that a locus is a geometrical figure such that any point in the figure and no point outside the figure fulfills the conditions stated. The class, then, plots a preliminary figure by placing several points or lines so that they fulfill the given conditions. They are shown that the limiting positions under the conditions are helpful. A good problem to illustrate the last statement was mentioned in the last paper. A parallelogram ABCD has the vertex A fixed and the directions of the adjacent sides AB and AD also fixed. Find the locus of the vertex C if the sum of the two sides AB and AD is constant, and prove your answer correct.

C′ and C″ suggest the assumption that fix the locus as a straight line, and the class can draw a second figure and proceed with the proof just as in any other proposition.

Concerning problems on the entrance papers may I ask a question or two?

Is the subject important enough to take one third of the paper as was done in June, '07?

Is it essential to force pupils working under stress of examination to take the time to discuss loci problems combining two simple problems so that a number of cases exist?

General Discussion.

Referring to Professor James Wilson's condemnation of the proof of the second part of locus theorems which involve the use of the *reductio ad absurdum*, Professor Osgood said that this method of proof was not to be rashly excluded. It was a valid method of proof, and was of use to mathematicians, and consequently must be of use to pupils.

Mr. Tower said that in his opinion Professor Osgood was wrong in assuming that what was of value to mathematicians was also good for pupils. In his opinion, pupils did not understand the *reductio ad absurdum*.

Professor Osgood said that that sort of information was just what he was glad to get. It was, however, important not to exclude the *reductio ad absurdum* on the ground that there was anything bad about it as a proof.

Mr. Fuller, on the other hand, claimed that boys were attracted by it.

Professor Wilson then said: Since this is my "slam," I suppose that I ought to justify it. I find that college boys do not use it intelligently. We are coached for so much, that our purposes in examination are defeated.

Mr. Fuller expressed his regret for the atmosphere of melancholy that seemed to weigh upon the college instructors present at the

meeting. He assured them of the sympathy of the secondary school teachers in their sorrowful mood; and expressed the opinion that it was not the subject of geometry which was required for admission to college, but the power of passing examination papers; he suggested that, if the examination requirement was so objectionable to everybody concerned, it would be quite possible to abolish it.

Mr. Hobbs pointed out that the proof by *reductio ad absurdum* could generally be replaced by the argument by exclusion; that is, of two possible conclusions, one of which must be true, if we prove one untrue, the proof of the other is inferred; and the same way for three possible conclusions, as in the case where one line must be equal to or greater than or less than another line.

Professor Ransom said that the Association spends too much time in talking about examinations. The locus problem he considered a test of extraordinary value. The definition that is given to beginners is not necessarily satisfactory to critics; nevertheless, a first definition serves a valuable purpose, though it may not be such as a mature and cultivated judgment would approve. Good training for developing the idea of a locus in the pupil's mind is to have him draw free-hand, so that the pencil-point shall keep true to the conditions of the locus. Professor Ransom also suggested that it was a good idea to bring back the idea of time into geometry.

Professor Lennes expressed surprise at the attention paid to college examination papers. The question of transfer from high school to college was, of course, important, but the question of transfer from the grade school to high school was also important. He thought we might well hesitate to impose the standards of the upper grades upon lower school work.

Professor Osgood said that he had been struck by the difficulty of getting pupils to draw. Most locus problems are easy to prove if the locus is drawn; and there is a great advantage in drawing the locus in advance, not only for the sake of clearing up the nature of the problem to be proved, but also for the sake of giving a mental picture of the effect of the restrictions imposed upon the moving point. However, not merely the drawing of the locus is desirable, but the proof also. The locus problem is one application of geometry that pupils can know. It may be that it is too hard for some weaker pupils: that is the teacher's work—to shelter those weak pupils, while giving real work to the pupils that have strength. It is the work of the teacher to choose judiciously the means of instruction, and the locus problem is a valuable means in the hands of a skillful teacher. In the same way that all complications are introduced rather by illustration than by definition, it would be better to begin teaching loci by showing a locus and explaining how to satisfy the conditions involved, and then to lead up to a definition which the pupils themselves could be induced to give. Professor Osgood suggested that such classical books of reference as Mr. Fuller has named should be in every school library, and he thought it quite likely that there were very few school libraries in which these books were represented.

THE TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

SYLLABUS.

ALGEBRA: ELEMENTARY AND INTERMEDIATE.

Report of the Committee.

LIST OF TOPICS.

(NOTE.—In this list no suggestion of order of topics is intended.)

I. Extension of Arithmetic in Algebra. Positive and Negative Numbers. Definitions. Graphs.
II. Fundamental Operations.
III. Factoring.
IV. Highest Common Factor and Least Common Multiple by factors.
V. Fractions; reduction, addition, subtraction, multiplication and division. Complex Fractions.
VI. Equations of the first degree in one unknown. Problems.
VII. Simultaneous Equations in two and three unknowns. Graphs. Problems.
VIII. Involution and Evolution. Square root of polynomials and arithmetical numbers.
IX. Exponents and Radicals. Radical Equations.
X. Imaginaries.
XI. Quadratic Equations in One and Several Unknowns. Theory. Graphs. Problems.
XII. Binomial Theorem for Positive Integral Exponent.
XIII. Inequalities.
XIV. Ratio and Proportion.
XV. Progressions.

FOREWORD.

This syllabus is intended primarily for teachers. It specifies those topics which, in the opinion of the Association of Teachers of Mathematics in the Middle States and Maryland, should be included under the designation, *Elementary and Intermediate Algebra.*

This association conceives the immediate purpose of the teaching of algebra in the schools to be the cultivation of the student's faculty for reasoning by helping him

(1) To concentrate his mind, especially on a *system* of thought;

(2) To generalize correctly; and,

(3) To develop originality and insight by using skillfully a finely worked out language of symbols.

It is believed that the body of closely related truths in algebra are admirably adapted to the purpose just mentioned. This is not to say that examples and illustrations to provide interesting

exercise in algebraic practice are not to be drawn from commerce, geometry, physics, mechanics; but such examples should be so simple as to require no extended explanation of their nature and whatever knowledge they imply should be regarded as incidental to the main purpose of cultivating the student's powers of reasoning. The first concern of the young student of algebra is the knowledge, logic, and operations contained within algebra itself, but it is perfectly possible to select attractive exercises which, while they do not carry the student far afield, will show him how algebra can be practically applied.

As the same idea occurs in different forms in various parts of the algebra, some repetition is unavoidable in the syllabus, but perhaps it is most desirable to recur several times to a method that does not change though it be applied to a form that is new.

Throughout the syllabus are notes bearing upon particular points, and at the close are appended a few general notes which are suggestive of the many observations the teacher has to make and of the caution his work constantly requires.

COMMITTEE:

GUSTAVE LEGRAS, The College of the City of New York, Chairman.
E. R. SMITH, Polytechnic Preparatory School, Brooklyn, New York.
LAO G. SIMONS, Normal College, New York.
G. A. SNOOK, Central High School, Philadelphia.
E. C. LAVERS, Easton High School, Easton, Pa.

December 4th, 1909.

The Committee heartily acknowledges its indebtedness to Prof. Jackson of Haverford College, to Mr. H. F. Hart of the Montclair High School, and to Dr. M. Philip, Mr. E. E. Whitford, and Mr. R. F. Smith of The College of the City of New York, for the valuable assistance these gentlemen gave the Committee in the preparation of this Syllabus.

I. EXTENSION OF ARITHMETIC IN ALGEBRA.

A. Literal numbers as the generalization of arithmetic numbers.

1. Indicated operations, $a+b$; $a-b$; $a\times b$; $a\div b$.
2. Powers and fractions result from indicated multiplications and divisions.
3. Minus numbers necessary for a more complete scale of numbers. In arithmetic 3—10 is an impossible operation. It becomes possible as soon as minus numbers are admitted. Introduce the scale...... —3, —2, —1, 0, +1, +2, +3, ...by addition and subtraction. Illustrate by divisions on a line, and by as many concrete examples as possible.

4. Simple problems involving the use of literal numbers, *e. g.*, John has a cents and received b cents, he spends x cents; how much has he? A letter stands for a number which may be integral, fractional, positive, or negative.

B. Simple equations.

1. Contrast 30% of cost=\$60 with $.3x=60$.
   $5+7=12$, $19-3=22-6$; substitute a letter in these examples.
2. Solution of $3x-4=x+8$ by the use of the equality axioms.
3. Discover law of transposition.
4. Literal equations.
   a. $x+a=b$
   b. $ax+b=c$
   c. More easy problems.
5. Some very simple problems resulting in numerical simultaneous equations.

Note 1. Extract definitions as they are needed.

Note 2. Graphs of simple forms such as $y=2x+1$, and $\begin{cases} y=2x+1 \\ y=3x+2 \end{cases}$
Use coördinate paper. Measure the x and y of intersection.

II. FUNDAMENTAL OPERATIONS.

NOTE. Introduce the laws of signs and exponents and the laws of commutation, association, and distribution as applied to these operations.

Some work in detached coefficients should be given.

A. Addition and Subtraction.

1. Algebraic addition involves arithmetic addition and subtraction.
2. Meaning of subtraction is to find a number which, added to the subtrahend, gives the minuend.
3. Removal and introduction of signs of aggregation. Simple cases only. Check by addition.
   Illustrative of the laws of addition.

B. Multiplication.

1. Monomials by monomials.
2. Polynomials by monomials.
3. Polynomials by polynomials.
   NOTE 1. Notion of *function* and *variable* may be suggested in this place by the evaluation of a polynomial for values of the letter in it.
   NOTE 2. Laws of homogeneity may be pointed out.
4. Type products.
   a. $(x\pm y)^2$, and the square of any polynomial.
   b. $(x+y)(x-y)$, $(x+y+z)(x+y-z)$, $(x^2+xy+y^2)(x^2-xy+y^2)$
   c. $(cx^2+a)(cx+b)$
   d. $(a\pm x)^3$
   e. Of the form, $(a\pm b)(a^4\mp a^3b+a^2b^2 \ldots\ldots\ldots)$.

NOTE.—It should be pointed out that the given expressions are the factors of the results.

C. Division.

1. Monomial divisors.
2. Polynomial divisors.
3. Special quotients of the type: $\frac{(x^n \pm y^n)}{x \pm y}$,

NOTE 1. Binomial divisors and simple synthetic division are especially recommended at this point.

NOTE 2. The Remainder Theorem should be introduced here by simple numerical examples. It may again be referred to in connection with factoring.
The simple proof by the equation $f(x) = Q(x-a)+R$ may well be given here.

III. FACTORING.

NOTE. Refer to II B, 4 and II C, 3.

1. Common Factor.
   a. $ax+bx$.
   b. $a(x+y)+b(x+y)$
   c. Grouping as $ax+ay+bx+by$.
2. Perfect square, $x^2 \pm 2ax+a^2$.
3. Difference of squares, x^2-a^2, $x^2 \pm 2ax+a^2-b^2$, $x^4+x^2y^2+y^4 = (x^4+2x^2y^2+y^4)-x^2y^2$
4. Quadratic trinomial, x^2+ax+b,
   $$ax^2+bx+c = \frac{a^2x^2+b(ax)+ac}{a}$$
5. Perfect cube, $x^3 \pm 3x^2y+3xy^2 \pm y^3$
6. Sum or difference of odd powers, $x^3 \pm y^3$. (Refer to Division 3.)
7. Binomial factors by the remainder theorem or synthetic division. Simple cases only.

NOTE 1. Above are the type forms to be lead up to by many simple numerical examples.

NOTE 2. Equations that can be solved by factoring as $x^2-3x=10$, $ax+bx+c$ should be introduced here.

IV. HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE.

A. Monomials only:
$60a^2x^2$, $45a^4x^3$, $20ax^4$.

B. Polynomials only:
(1) x^3-8, x^2-4, $x^2+5x-14$.
(2) $x^2-ax-bx+ab$, $x^2-2bx+b^2$, x^2-b^2.

C. Products of monomial and polynomial.
$10a^2x^2-90a^2$, $15a^3x^2+30a^2x-225a^3$.

V. Fractions.

Note.—A whole number is a fraction with the denominator 1.

A. Reduction to lowest terms.

1. Monomial numerator and denominator:

$$\frac{49a^5x^3}{35a^7x^2}=\frac{7.7.a^5.x^2.x}{7.5.a^5.a^2.x^2}=\frac{7.a^5.x^2}{7.a^5.x^2}.\frac{7x}{5a^2}=$$ Then do again by cancellation.

2. Polynomial numerator and denominator:

a. Type, $\frac{(x-7)(x+7)}{(x-7)(x-5)}$ or $\frac{(x-a)(x+b)}{5(x-a)}$ etc.

b. Type, $\frac{(x-5)(x+3)}{a(5-x)}$ or $\frac{(x-a)(x-b)}{(b-x)(b+x)}$

Note.—What is cancellation? Emphasize $a-a=0$, $\frac{a}{a}=1$, $\frac{a-b}{a-b}=?$

B. Reduction to mixed expressions and the reverse. Many examples, especially of the latter.

C. Addition and subtraction of fractions.

1. Reducing fractions to a common denominator.
   a. Monomial.
   b. Binomial and polynomial.
2. Addition and subtraction.
   a. Monomial denominators.
   b. Polynomial denominators.

1. Type, $\frac{m+n}{m-n}-\frac{m-n}{m+n}-\frac{4m^2}{m^2-n^2}$

2. Type, $\frac{b+c}{(a-b)(a-c)}+\frac{c+a}{(b-c)(b-a)}+\frac{a+b}{(c-a)(c-b)}$

Note.—Examples, in which the sum can be reduced to lower terms, are especially recommended.

D. Multiplication and division of fractions.

1. Monomial numerators and denominators.
2. Polynomial numerators and denominators.

Note 1.—Arithmetical processes with fractions should now be recalled and compared with the corresponding operations in algebra, such as multiplying and dividing both terms by the same number, reducing to the same common denominator, etc. For each of these processes a sufficient reason should be given and appeal made to the student's intuition.

Note. 2.—The first elementary notion of ratio may be introduced with fractions and the forms $\frac{a}{\infty}$, $\frac{0}{a}$, $\frac{a}{0}$, $\frac{0}{0}$, explained with such simple numerical examples as, $\frac{2}{\infty}$, $\frac{0}{2}$, $\frac{2}{0}$, $\frac{x^2-1}{x-1}$, when $x=1$.

E. Simplification of complex fractions.

1. The continued fraction (simple types only).

$$\frac{1}{1+\frac{1}{x+\frac{1}{x}}}$$

2. Complex fraction having a sum or difference in both numerator and denominator.

$$\frac{\frac{1}{a-b}-\frac{a}{a^2-b^2}}{\frac{a}{ab+b^2}-\frac{b}{a^2+ab}}$$

3. Mixed numbers in both numerator and denominator.

$$\frac{\frac{2mn}{m+n}-1}{1-\frac{n}{m+n}}, \quad \frac{2x-1-\frac{10}{x+3}}{3x-5+\frac{12}{x-2}}.$$

NOTE.—Multiplying both terms of the fraction by the same expression is often the simplest device. Examples should not be too complex and some should be workable in several ways.

VI. THE SIMPLE EQUATION.

A. Extended practice in the simple equation, resuming the consideration of transposition.

B. The fractional equation:

1. Monomial denominators:

$$\frac{x-1}{7}=7-\frac{4+x}{4}-\frac{23-x}{5}$$

2. Polynomial denominators:

a. $$\frac{2x+1}{3x-3}=\frac{7x+1}{6x-6}-\frac{2x^2-3x-45}{4x^2-4}$$

b. $$\frac{1}{x-1}-\frac{1}{x-2}=\frac{1}{x-3}-\frac{1}{x-4}$$

NOTE.—Simplify the members separately before clearing of fractions in *b.*

c. $$\frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}$$

C. The literal equation:

1. Solution for one unknown.

2. "Formula work." Solving simple formulæ for each of the letters in terms of the others.

Note.—Only formulæ that can be *briefly* explained should be chosen, as:

(*a*) $i=\frac{ptr}{100}$, (simple interest)

(*b*) $d^2=2s^2$, (diagonal of a square)

(*c*) $h=\frac{s}{2}\sqrt{3}$, (height of equilateral triangle)

(*d*) $C=(F-32)\frac{5}{9}$, (Centigrade scale)

(*e*) $s=vt$
$c=2\pi r$

$i=\frac{l}{d^2}$, (light), etc., giving numerical illustrations.

D. Problems leading to simple equations.

1. Oral work, as: the sum of a and b=? When a is divided by b, c is the quotient, and d the remainder; express algebraically. A train goes at the rate of m miles an hour for h hours, how far does it go? The interest on a dollars at b per cent for c years is what? And so on.
2. Problems dealing with—
   - *a.* Geometrical objects.
   - *b.* Motion (trains, etc.)
   - *c.* Number.
   - *d.* Business.
3. Some simple abstract problems, illustrated by concrete examples.

VII. Equations in Two and Three Unknowns.

A. Two unknowns.

Elimination by, 1. Addition and subtraction.
2. Comparison.
3. Substitution.

Note.—Some problems should be done by all three methods for the sake of comparison.

B. Two unknowns, fractional form:

$$\frac{4}{5x}+\frac{5}{6y}=5\frac{11}{15}$$

$$\frac{5}{4x}-\frac{4}{5y}=\frac{11}{20}$$

C. Three unknowns.

1. Where all three equations contain all three unknowns, with small numerical coefficients only.
2. Where the three unknowns are not in each equation.

$$\frac{4}{x}-\frac{3}{y}=\frac{1}{20}$$

$$\frac{2}{z}-\frac{3}{x}=\frac{1}{15}$$

$$\frac{4}{z}-\frac{5}{y}=\frac{1}{12}$$

D. Literal equations.

$$(a-b)x-(a+b)y=-4ab$$
$$(a+b)x+(a-b)y=2a^2-2b^2$$

E. Problems leading to equations in two and three unknowns.

F. Plotting easy equations in two unknowns, such examples as give:

(1) Lines intersecting at any angle.

(2) At right angles.

(3) Lines that are parallel, pointing out the relations between the coefficients.

VIII. Involution and Evolution.

A. Powers of a monomial. The general form $(a^m)^n$ carefully examined and illustrated.

B. Square, cube and fourth power of a binomial.

C. Definition of root. Principal root. Roots of a monomial.

D. Square root of:

1. Polynomial presented through trinomial.
2. Numbers.

Note.—Derive the general method by the type form, $a^2+2ab+b^2$

IX. Exponents and Radicals.

A. Exponent in fundamental operations. (Refer to II).

B. Theory of a positive integral exponent.

Theorems:

1. $x^a \cdot x^b = x^{a+b}$
2. $x^a \div x^b = x^{a-b} \qquad (a>b)$
3. $(x^a)^b = x^{ab}$
4. $(x^a)^b = (x^b)^a$
5. $(x^a y^b)^n = x^{an}y^{bn}$
6. $\left(\frac{x^a}{y^b}\right)^n = \frac{x^{an}}{y^{bn}}$
7. $(\sqrt[n]{x^a})^n = x^a$

Note.—In proving the above theorems use numerical exponents, the exponents being the abbreviations of continued multiplication, and then I, Aa. Restrict a and b to positive integers, $a>b$.

C. Negative, zero, and fractional exponents:

Note.—Apply the principle of *no exception* to interpret these new exponential forms.

Theorems:

1. $\frac{a^m}{a^n}=\frac{a^{m-n}}{1}=\frac{1}{a^{n-m}}=\frac{1}{a^{-(m-n)}}$

The scale. ......a^{-4}, a^{-3}, a^{-2}, a^{-1}, a^0, a^1, a^2, a^3......

Work in fundamental operations with these exponents.

2. When $n=m$, $\frac{a^m}{a^n}=\frac{a^m}{a^m}=a^{m-m}=a^0=1$

3. *a* $x^{\frac{a}{n}}=\sqrt[n]{x^a}$ *cf*. B. 7

b $\sqrt[n]{x^a}\cdot\sqrt[n]{y^b}=\sqrt[n]{x^a\cdot y^b}$

NOTE.—All these theorems should be first indicated by numerical examples.

D. 1. Radicals closely associated with exponents wherever possible.

2. The fractional exponent and radical sign interchanged and compared in application.

3. The graphic representation of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$ by successive right-angled triangles.

E. Operations:

1. Removal of a perfect power: $\sqrt{18a^3b^5}=3ab^2\sqrt{2ab}$

2. Reduction to lower degree: $\sqrt[6]{9a^2b^4}=\sqrt[3]{3ab^2}$ or vice versa.

3. Introduction of a factor under the radical sign: $2a\sqrt[3]{3b}=\sqrt[3]{24a^3b}$

4. The four fundamental operations and powers.

$$2\sqrt{a}+3\sqrt{a}=5\sqrt{a}\,,\ \sqrt{a}\cdot\sqrt{b}\cdot\sqrt{c}=\sqrt{abc},\ \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}};$$

$(\sqrt{abc})^2$ etc. It is well to state some of these laws in words.

5. Rationalizing the denominator limited to monomials and binomials of the second order, including such examples as:

$$\sqrt{\frac{x}{y}},\ \frac{\sqrt{3}-x}{\sqrt{2}},\ \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}.$$

Find the approximate value of $\frac{\sqrt{5}-1}{3-2\sqrt{5}}$ to three decimal places, *after* simplification.

6. Radical equations. Such forms as:

$$\sqrt{2x+1}=2x-3$$
$$\sqrt{x-1}+\sqrt{x+1}=\sqrt{x-2}$$
$$\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$$
$$\frac{5x-1}{\sqrt{5x}+1}=1+\frac{\sqrt{5x}-1}{2}$$

X. IMAGINARIES.

NOTE.—On entering upon the subject of imaginaries, it should be pointed out that negative numbers and fractions are not *real* in the sense that numbers like 5 and $\frac{24}{6}$ are real, and hence in imaginaries the student is again extending his number system.

A. Definition equations.

1. $(\sqrt[n]{-a})^n = -a$

Corollaries:

a $\sqrt{-a}=\sqrt{a}\sqrt{-1}$

b $\sqrt{-a}\sqrt{-b}=-\sqrt{ab}$

2. $\sqrt{-1}=i$

1. Corollaries:

$a.$ $i^{4q+1}=1$

$b.$ $i^{4q+2}=-1$

$c.$ $i^{4q+3}=-i$

$d.$ $i^{4q+4}=+1$

NOTE.—Begin with $q=0$, 1, 2, etc. The constant use of the symbol i is strongly recommended.

B. Complex numbers.

1. Definitions.
2. Theorems.

a. The sum and product of two conjugate complex numbers are both real.

b. If $X+iY=0$, then $X=0$ and $Y=0$

Corollary:

If $X+iY=A+iB$, then $X=A$ and $Y=B$.

3. The fundamental operations.

a. Addition and subtraction.

b. Multiplication and division.

Rationalization of denominators.

Conjugates.

C. Graphs of complex numbers; their sums and differences.

XI. QUADRATIC EQUATIONS IN ONE AND SEVERAL UNKNOWNS. THEORY.

A. Equations in one variable.

1. Solution.

a. Incomplete.

(1) Pure, reducible to the form $x^2=a$, $x=\pm a$;

or $x^2-a=0$, $(x+\sqrt{a})(x-\sqrt{a})=0$, etc.

(2) Lacking the independent term, of form $ax^2+bx=0$. Solved by factoring.

b. Complete, reducible to the form $ax^2+bx+c=0$

Solved by *a.* Factoring.

b. Completing the square.

c. Quadratic formula.

NOTE.—Fractional and radical equations which reduce to these type forms are included. Care should be taken that clearing of fractions or clearing of radicals does not introduce a false root.

c. Equations which are quadratic in a power of the variable or any function of it.

(1) In a power, as $ax^{2n} + bx^n + c=0$

or $ax^{\frac{m}{n}}+bx^{\frac{m}{2n}}+c=0$

(2) In any function, as

$$(x^2+2x)^2-2(x^2+2x)-3=0,$$

$$2x^2-6x+\sqrt{x^2-3x+6}-9=0,$$

$$\frac{x^2}{x+1}+\frac{x+1}{x^2}=\frac{5}{2}.$$

Solved by factoring, or by substituting a new variable for the function involved.

d. Equations of higher degree which factor by known methods, especially by synthetic division.

B. Theory.

1. Proof of the quadratic formula.
2. Symmetric functions of the roots:
   (1) Sum and product of the roots; application to testing results and to forming equations from given roots. Simple symmetric functions, as $\frac{1}{r}+\frac{1}{s}$, where r and s are the roots.
   (2) Use of the product to show the roots are reciprocal, if the coefficient of the second degree term equals the independent term.
3. Discriminant test for the roots:
   (1) Real or imaginary.
   (2) Rational or irrational.
   (3) Equal or unequal.

NOTE.—Applications such as the following are recommended: For what values of a will the roots of $2x^2+(1+a)x+2=0$ be equal? Real?

3. Graphs. Type form, ax^2+bx+c.
   a. Let $ax^2+bx+c=y$.
   b. For approximation of the roots, change to form x^2+px+q, let $y=x^2$ and $y+px+q=0$. Plot the intersections of the curve and the straight line. The curve $y=x^2$ is convenient for all numerical quadratic equations.
4. Problems including numerical, geometrical, physical and commercial relations, provided their subject matter does not require extended explanation.

B. Equations in two variables which can be solved by quadratic methods.

1. Solution.
   a. When one is of the first degree, the other of second degree or factorable by the simple equation so as to reduce to second degree.
   Solved by substitution, as
   $ax+by=c$
   $dx^2+exy+fx=k$, etc., etc.
   b. Homogeneous second degree equations.
   1. One entirely homogeneous, the other not, as
   $ax^2+bxy+cy^2=0$
   $lx^2+y+my^2=k$
   Solved by factoring, then substituting.

2. Both homogeneous except for the independent term, as
$ax^2+bxy+cy^2=d$
$ex^2+fxy+gy^2=k$
Solved by eliminating the independent term, then treating like case (a.)

c. Symmetric forms (or symmetric but for sign)
Solved by obtaining $x+y$ and $x-y$
1. Of types $x+y$, $x-y$, xy, x^2+y^2, $x^2\pm xy+y^2$, $x^4+x^2y^2+y^4$, $\dfrac{x}{y}+\dfrac{y}{x}$
2. Of type $(ax)^n+(by)^n=k$
$ax+by=1$
Solved by elimination of the powers. Divide if possible.

d. Other methods.
1. Divide one equation by the other.
(a) When it divides evenly, as
$x^3+y^3=28$
$x+y=4$
(b) When a like factor appears in a member of each equation, as
$a\ (x^2-y^2)=b$
$c\ (x-y)=d$
2. Add or subtract:
(a) When one unknown is thus eliminated, as
$x^2+y=7$
$x^2-x-y=1$
(b) When the result is a factorable form, as
$x^2+xy+y^2+x+y=10$
$xy+2x+2y=8$
(c) Factor when possible, as
$x^2y^2\pm 5xy+4=0$
$x^2+y^2-xy=13$

e. Equations of the foregoing types which are in terms of functions of the variables, as
$(x+y)^2+x^2y^2=13$
$x+y-xy=1$
Solve for the functions (here, for $x+y$ and xy).
Such forms may be simplified by the substitution of a new variable.

2. Graphs.
Of any equation in two variables.
Approximation of results for cases which do not solve by quadratic methods. Simple cases only.

3. Problems.
Resulting in equations of the foregoing types.

C. Equations in three or more variables.
Simple types only.

XII. Binomial Theorem for Positive Integral Exponents.

A. Theory.

1. Proof of expansion of $(a+b)^n$ by simple inductive reasoning.
2. Discussion of the formula.
   - *a.* As a whole.
     - (1) Number of terms.
     - (2) Sum of the coefficients of $(a+b)^n=(1+1)^n$; of $(3a+4b)^n=(3+4)^n$.
     - (3) Equality of coefficients of terms equally distant from the extremes.
   - *b.* The general term.
     - (1) Form of the coefficient in the fractional factored form.
     - (2) Sum of exponents in any term.

B. Applications.

1. To expansion of any binomial.
2. To finding any term, by formula or by analogy say with the 3d term.
3. Such problems as, (1) Find coefficients of x^{12} in $(x^2+2x)^{10}$
   (2) Find term independent of x in
   $$\left(x^2-\frac{1}{x}\right)^{12}$$
   (3) Find middle term of $\left(2-\frac{x}{3}\right)^8$.
4. Powers of numbers, as $(1.1)^{12}=(1+.1)^{12}$ correct to 3 decimal places.
5. Expansion of some simple polynomials.

XIII. Inequalities.

I. Proofs of the following theorems:

A. Unequals combined with equals.

Note.—Prove by equation methods, *i. e.*, if a is greater than b, let $a=b+x$, where x is a positive quantity, and proceed by the equality axioms.

1. *a.* Equals added to unequals give unequals in the same sense.
   b. Equals taken from unequals give unequals in the same sense.
   c. A term can be changed from one member of an inequality to the other if the sign of the term be changed; application to annulment of like terms in both members.
   d. The signs of all terms of an inequality can be changed if the inequality sign is reversed.
2. *a.* Positive equals multiplied into unequals give unequals in the same sense.
   b. Positive equals divided into unequals give unequals in the same sense.

3. *a.* Unequals taken from equals give results unequal in the opposite sense.
   b. Positive unequals divided into positive equals give results unequal in the opposite sense.

B. Unequals combined with unequals.
   1. Unequals added to unequals in the same sense give results in that sense.
   2. *a.* Positive unequals multiplied by positive unequals in the same sense give results unequal in that sense.
      b. Positive unequals raised to a positive integral power give results unequal in the same sense.
      c. Positive unequals raised to a positive fractional power give results unequal in the same sense.
      d. Positive unequals raised to a negative power give results unequal in the opposite sense: reciprocals.

   NOTE.—Unequals should not be taken from or divided by unequals, for the results can not in general be determined.

II. Solutions.

A. One unknown
   2. Of type $ax^2+bx+c>0$
   3. Of type $\dfrac{(x-a)\ (x-b)}{(x-c)\ (x-d)} \gtrless 0$

B. Two unknowns
   1. Of type $3x+5y>60$
      $3x+2y=30$
   2. Of type $x-2y+1<0$
      $x+\ y-5<0$
      $2x-y-1>0$

III. Use of the theorems with the fact that a perfect square cannot be negative, to prove, (a and b being positive unequal numbers), $a^2+b^2>2ab$, $a^3+b^3>a^2b+ab^2$

XIV. RATIO AND PROPORTION.

I. Proofs of the following theorems:

A. 1. The product of the means equals the product of the extremes.
   2. In a mean proportion, the mean equals the square root of the product of the extremes.
   3. Either mean equals the product of the extremes divided by the other mean, and either extreme equals the product of the means divided by the other extreme.

B. 1. If the product of two quantities equals the product of two other quantities, either pair may be made the means, and the other pair the extremes, of a proportion.
   2. If four quantities are in proportion, any proportion of these quantities is true if the means of the original proportion are either both means or both extremes, in the new proportion.

C. 1. If four quantities are in proportion, they are in proportion by composition.

2. If four quantities are in proportion, they are in proportion by division.
3. If four quantities are in proportion, they are in proportion by composition and division.

D. 1. If four quantities are in proportion, equi-multiples (integral or fractional) of the antecedents are in proportion to equi-multiples of the consequents.
2. If four quantities are in proportion, like powers of those quantities are in proportion.

E. If a series of ratios are equal, the ratio of the sum of their antecedents to the sum of their consequents equals any one of those ratios.

II. Applications.
A. Deduction of any proportion from a related proportion.
B. Finding one term if the others are known.
C. Solving equations (usually by composition and division.)

XV. PROGRESSIONS.

I. Arithmetic.
A. Deduction of formulas for
1. General or last term.
2. Sum of any number of terms.
3. Mean between two quantities.

B. Applications.
1. Given any three of the five numbers, first term, difference, number of terms, last term, sum, to find the other two.
2. Insertion of any number of means between two quantities.
3. Problems of other types which can be solved by use of the formulas.

II. Geometric.
A. Deduction of formulas for
1. General or last term.
2. *a*. Sum of any number of terms.
b. Limit approached by the sum of an infinite decreasing series.
3. Mean between two quantities.

B. Applications.
1. Given any three of the five numbers, first term, ratio, number of terms, last term, sum, to find the other two, unless the equations cannot be solved by known methods.
2. Insertion of any number of means between two quantities.
3. Evaluation of recurring decimals.
4. Problems of other types which can be solved by use of the formulas, some combining the two progressions and some reviewing important types of quadratics.

GENERAL NOTES.

1. All the topics in this syllabus may at first be studied in outline with simple examples and then reviewed in detail with harder examples.

2. There should be much practice in mental algebra. Students should learn to give the elementary, simple forms accurately and quickly.

3. Students should state principles in general language without symbols and without writing; at other times the general principle should be given them and they should be required to translate it promptly into symbolic language.

4. The definition should be given at the moment that it is required. It should be more often evoked than imposed. The student may be allowed to make and to correct his own statement. A temporary definition may be allowed until such time as correction and enlargement are necessary.

5. Home examples should relate generally to what has already been studied in class. The student should be encouraged to select for himself examples belonging to any advance work he is preparing. Sometimes a special topic in which a student has made commendable progress may be assigned to that student for further work which he may explain to the class.

6. Clear, orderly expression, writing and arrangement should be exacted at all times. The student should be made to feel the disadvantage of disorderly arrangement in expression.

7. Guessing in answering should be discouraged. A student should only say what at least he believes to be correct, and he should not lay too much emphasis on answers merely as answers. He should distinguish between answers and roots in certain problems.

8. Great care should be taken that algebraic work does not become mechanical. It may be necessary to repeat many times the *meaning* of the various algebraic forms.

9. The equation is the chief concern of algebra. In performing the various operations on equations great care should be taken with signs, with interchange of members, with simplifications sometimes before and sometimes after clearing of fractions, etc. Care should be taken to distinguish between unknown and variable. In the simple graphs of equations with one unknown, and of simultaneous equations with two unknowns, very clear distinction of the lines representing the values of the unknown or unknowns should be made.

10. In literal equations the meanings of the several kinds of letters used should be emphasized and generously illustrated by numerical values.

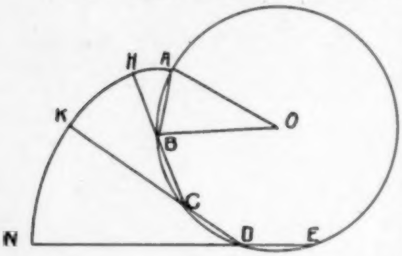
11. Operations should be frequently checked, preferably by numerical substitution wherever possible.

THE AREA AND LENGTH OF THE INVOLUTE OF A CIRCLE BY ELEMENTARY METHODS.

By G. D. L'HOMMEDE,

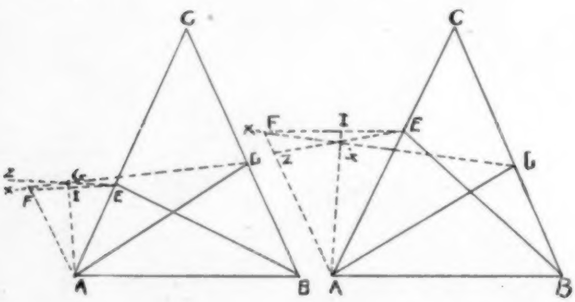
Los Angeles, Calif.

I. To find the area contained between a circle and its involute.



Given the circle O, of radius r. Inscribe in the circle a regular polygon of m sides. Let AB, BC, CD, DE be sides of the polygon. With B as a center and radius BA describe an arc intersecting CB produced in H. With C as a center and radius CH describe an arc intersecting DC produced in K. With D as a center and radius DK describe an arc intersecting ED produced in N. Join AO, BO.

Angle $AOB=2\pi/m$. The angles ABH, HCK, KDN are all equal to $2\pi/m$. $AB=2r\sin\pi/m$. Since HC=2 AB, sector HCK=4 sector ABH; since DK=3 AB, sector KDN=9 sector ABH; and so on, the nth sector$=n^2$ sector ABH.



Call S the sum of all the sectors and we have,

$$S=(1+2^2+3^2+\ldots\ldots+m^2)\text{ sector ABH.}$$

$$=\frac{2m^3+3m^2+m}{6}\text{ sector ABH.}\quad(1.)$$

The area of sector $ABH=\dfrac{\pi AB^2}{m}=\dfrac{4\pi r^2\sin^2\pi/m}{m}$

Substituting in (1) we have,

$$S=\frac{2m^3+3m^2+m}{6}\cdot\frac{4\pi r^2\sin^2\pi/m}{m}$$

If m becomes very large and tends toward infinity, the line AHKN approaches the involute as its limiting position, the inscribed polygon approaches the circle as its limit, and the sum of the sectors approaches the area between the circle and the involute. In our last equation $\sin\pi/m$ becomes arc π/m and we have,

$$S = \frac{2m^3 + 3m^2 + m}{6} \cdot \frac{4\pi r^2 \pi^2/m^2.}{m}$$
$$= 4\pi^3 r^2 \left(\tfrac{1}{3} + \tfrac{1}{2}m + \tfrac{1}{6}m^2\right)$$

At the limit the last two terms in the parenthesis vanish and we have, $S = \frac{4\pi^3 r^2.}{3}$

If, instead of summing up all the sectors, we had summed up n sectors (n increasing with m so that the ratio of n to m remains finite) we would find the area $= \frac{4\pi^3 r^2}{3} \cdot \frac{n^3}{m^3}$. We see that the areas corresponding to different arcs are to each other as the cubes of the arcs. If we make $r=1$, we get $S = \frac{4\pi^3}{3} = \frac{8\pi^3}{6} = \frac{(2\pi)^3}{6}$. The total area equals one-sixth of the cube of the circumference. But the areas corresponding to different arcs being to each other as the cubes of the arcs, we can say that the area corresponding to any arc is equal to one-sixth the cube of the arc (π measure).

That is the formula obtained by the integral calculus.

II. For the rectification we follow a similar method.

Call L the sum of the arcs AH, HK, KN.......

$$L = AH\ (1+2+3+4+\ldots\ldots+m) = AH\ m\ (m+1)/2$$

$$AH = \frac{2\pi AB}{m} = \frac{4\pi r \sin \pi/m}{m}$$

$$L = \frac{4\pi r \sin \pi/m}{m} \cdot \frac{m\,(m+1)}{2}$$

$$\lim_{m=\infty} \left[L \right] = 2\pi^2 r.$$

For $r=1$, we have $L = 2\pi^2 = \frac{(2\pi)^2}{2}$

The length of the curve corresponding to the entire circle is one-half the square of the circumference (π measure).

We could show, by a method similar to that used for the area, that the length of curves corresponding to different arcs are to each other as the squares of the arcs; so we can say that the length of the involute corresponding to any arc is equal to one-half the square of the arc (π measure). This is the formula found by the integral calculus.

To reëstablish the radius from the π measure formulas we multiply by r^2 (not r^3) for the area, and by r (not r^2) for the length.

A NOTATION FOR THE GEOMETRY OF THE SPHERE.

HOWARD F. HART.

The writer in perfecting the proof of the theorem, "A spherical triangle equals a lune whose angle is half the spherical excess of the triangle," so that it would not seem so clumsy, discovered the notation given below. He further found that it not only simplified the proof of this proposition, but also that of others in the geometry of the sphere as well.

In this notation the spherical triangle of angles A, B, and C is denoted by $T_{A,B,C}$; the spherical polygon of angles $A_1, A_2 \dots\dots\dots A_n$ by $P_{A_1, A_2 \dots\dots A_n}$; the lune of angle A by L_A; etc.

Then we have, *e. g.*:

(1) $L_A \pm L_B = L_{A \pm B}$

(2) $\frac{1}{n} L_A = L_{A/n}$

(3) $\frac{L_A}{L_B} = \frac{A}{B}$

(4) $L_{360^\circ} = 4\pi R^2$ R = radius of sphere

(5) $T_{A,B,C} = \frac{L_{A+B+C-180^\circ}}{2}$ (which is the theorem quoted above)

(6) $P_{A_1, A_2 \dots A_n} = \frac{L_{(A_1 + A_2 + \dots + A_n) - (n-2)180^\circ}}{2}$

etc.

It will be observed that the notation is operative, which is probably the cause of its simplifying power and of its success as a class room device.

High School, Montclair, N. J.

NOTE ON "A DIRECT DEMONSTRATION."

WM. A. LUBY.

"A Direct Demonstration" in SCHOOL SCIENCE AND MATHEMATICS for May is incorrect. The error creeps in at the point where it is proved that AH=AG. It follows then that G and H are the same point, and that DX intersects EZ in that point. Call the intersection G and it follows that E is either on DG, above it or below it. To assume that E is on DG is to assume that the original theorem is true. Two other assumptions remain, giving the two possibilities illustrated in the figures below. In each of these figures angles IEG and IFG are not alternate interior angles. The conclusion that DX is parallel to EZ is therefore false.

Central High School, Kansas City, Mo.

PROBLEM DEPARTMENT.

E. L. Brown.

Principal North Side High School, Denver, Colo.

Readers of the magazine are invited to send solutions of the problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott St., Denver, Colo.

Algebra.

166. *Proposed by James A. Whitted, Abingdon, Ill.*

Four bonds of $4,000 each, bearing 6% interest, dated Sept. 1, 1895, and falling due in 1, 2, 3, 4 years respectively, are sold Oct. 19, 1895. What is the price that they should net the purchaser 5% per annum, compounded annually?

I. Solution by G. B. M. Zerr, Philadelphia, Pa.

The first bond has 10 mos. and 12 days to run; the second, 1 yr., 10 mos., 12 days; the third, 2 yrs., 10 mos., 12 days; the fourth, 3 yrs., 10 mos., 12 days.

Interest on $1 for 10 mos. 12 days at 5% = $0.04⅓.

Price of first bond $= \dfrac{\$4000 \times 1.06}{1.04\frac{1}{3}} = \$4063.898.$

Price of second bond $= \dfrac{\$4000 \times (1.06)^2}{1.05 \times 1.04\frac{1}{3}} = \$4102.602.$

Price of third bond $= \dfrac{\$4000 \times (1.06)^3}{(1.05)^2 \times 1.04\frac{1}{3}} = \$4141.674.$

Price of fourth bond $= \dfrac{\$4000 \times (1.06)^4}{(1.05)^3 \times 1.04\frac{1}{3}} = \$4181.119.$

Total = $16489.293.

II. Solution by W. T. Brewer, Quincy, Ill.

Amount of $1 for 10 mos., 12 days = $1.042

$4240 ÷ 1.042 = $4069.10, cost of first bond.

Amount of $1 for 1 yr., 10 mos., 12 days = $1.095.

$4480 ÷ $1.095 = 4091.32, cost of second bond.

Amount of $1 for 2 yrs., 10 mos., 12 days = $1.150

$4720 ÷ 1.150 = $4104.34, cost of third bond.

Amount of $1 for 3 yrs., 10 mos., 12 days = $1.207

$4960 ÷ 1.207 = $4109.36, cost of fourth bond.

Total cost, $16374.11

III. Solution by Orville Price, Denver, Colo.

The amounts of the four bonds, at simple interest, are $4240, $4480, $4720, $4960, a total of $18400.

The amount of $1 for 3 yrs., 10 mos., 12 days, at 5% per annum, compounded annually, is $1.2077.

$18400 ÷ 1.2077 = $15235.407, price to be paid.

Also solved with different result by W. B. Borgers. Different views being taken of such problems, we can not expect uniform results.

167. *Proposed by H. E. Cobb, Chicago, Ill.*

The first three terms of a series are 1, 7, 19; and the general term, U_n, is an integral function of the second degree in n. Find the general term and the sum of the first n terms.

I. Solution by John P. Clark, Asbury Park, N. J.

Let $U_n = xn^2 + yn + z$.
For $n=1$, $x + y + z = 1$;
For $n=2$, $4x + 2y + z = 7$;
For $n=3$, $9x + 3y + z = 19$.
Solving, $x=3$, $y=-3$, $z=1$.
Hence $U_n = 3n^2 - 3n + 1$.
Therefore the series is 1, 7, 19, ...$3n^2 - 3n + 1$.
First order of differences, 6, 12, 18, 24, ...
Second order of differences, 6, 6, 6, ...
Third order of differences, 0, 0, ...

$S_n = na_1 + \frac{n(n-1)}{\lfloor 2} d_1 + \frac{n(n-1)(n-2)}{\lfloor 3} d_2 + \ldots$, where $a_1=1$, $d_1=6$, $d_2=6$, $d_3=0, \ldots$

Substituting and combining we get $S_n = n^3$.

II. Solution by W. B. Borgers, Grand Rapids, Mich. and T. M. Blakslee, Ames, Iowa.

As in solution I, $U_n = 3n^2 - 3n + 1 = n^3 - (n-1)^3$.
$S_n = [n^3-(n-1)^3]+[(n-1)^3-(n-2)^3]+[(n-2)^3-(n-3)^3]+\ldots\ldots = n^3$.

III. Solution by the Proposer.

Let $1+7+19 \ldots\ldots (3n^2-3n+1) = A+Bn+Cn^2+Dn^3+\ldots$
Then $1+7+19 \ldots\ldots [3(n+1)^2-3(n+1)+1] = A+B(n+1)+C(n+1)^2+D(n+1)^3+\ldots$
Subtracting, $3n^2+3n+1 = B+C(2n+1)+D(3n^2+3n+1)+\ldots$
Equating coefficients and solving, we have
$D=1$, $B=0$, $C=0$,
$\therefore 1+7+19+\ldots\ldots(3n^2-3n+1) = A+n^3$
Putting $n=1$, we have $A=0$.
Therefore the sum of n terms is n^3.

168. *Proposed by I. L. Winckler, Middlebury, Vt.*

Solve $(y+z)(x+y+z)=a \ldots\ldots (1)$
$(z+x)(x+y+z)=b \ldots\ldots (2)$
$(x+y)(x+y+z)=c \ldots\ldots (3)$

Solution by Eugene R. Smith, Brooklyn, N. Y.

Adding equations (1), (2), and (3), and letting $a+b+c=2s$, we get $(x+y+z)^2=s$, or $x+y+z=\sqrt{s} \ldots\ldots(4)$.

Dividing (4) into (1) and subtracting the result from (4), we get $x=\frac{s-a}{\sqrt{s}}$, where $s=\sqrt{\tfrac{1}{2}(a+b+c)}$.

By symmetry, $y=\frac{s-b}{\sqrt{s}}$, and $z\frac{s-c}{\sqrt{s}}$.

Geometry.

169. *Proposed by B. C. Eastham, Salem, Oregon.*

Given four sides of a quadrilateral and another line, construct the quadrilateral so that its area will be equivalent to the square on the fifth line.

Solution by G. B. M. Zerr, Philadelphia, Pa.

Let a, b, c, d be the sides, and A, B, C, D the angles of the quadrilateral, x the diagonal BD, e the fifth line.

Then $ad \sin A + bc \sin C = 2e^2 \ldots\ldots (1)$

$x^2 = a^2 + d^2 - 2ad \cos A = b^2 + c^2 - 2bc \cos C$

$\therefore ad \cos A - bc \cos C = \frac{1}{2}(a^2 + d^2 - b^2 - c^2) = p \ldots\ldots (2)$

From (1), $bc \sin C = 2e^2 - ad \sin A \ldots\ldots (3)$

From (2), $bc \cos C = ad \cos A - p \ldots\ldots (4)$

Squaring (1) and (2), and adding, we get $4e^2 \sin A + 2p \cos A = \frac{1}{ab}(p^2 + 4e^4 + a^2d^2 - b^2c^2)$.

Hence A and BD $= x$ are known. Then with D as a center and radii d, c, describe arcs; also with B as a center and radii a, b, describe arcs; d, a intersect at A, b and c at C.

An elegant geometric solution of this problem is found on pp. 160-161 in Casey's Sequel to Euclid.

170. *Proposed by H. C. Whitaker, Philadelphia, Pa.*

There is a loaf which is a half sphere of radius r. How thick is the crust baked if the soft part is one nth of the whole loaf?

Solution by G. B. M. Zerr, Philadelphia, Pa.; and J. H. Weaver, Plain City, Ohio.

Let x = thickness of crust. Then the soft part is a segment of a sphere of radius $r-x$; $r-2x$ = height of segment.

Volume of loaf $= \frac{2\pi r^3}{3}$.

$$\text{Volume of segment} = \pi (r-2x)^2 [(r-x) - \tfrac{1}{3}(r-2x)]$$
$$= \tfrac{1}{3}\pi (2r-x)(r-2x)^2.$$

$$\therefore \tfrac{1}{3}\pi (2r-x)(r-2x)^2 = \frac{2\pi r^3}{3n}$$

$$\therefore x^3 - 3rx^2 + \tfrac{9}{4}r^2x = \frac{(n-1)r^3}{2n}. \quad \text{Let } x = y + r.$$

$$\text{Then, } y^3 - \tfrac{3}{4}yr^2 = \frac{(n-2)r^3}{4n}. \quad \text{Let } \sin\theta = \frac{2-n}{n}.$$

Therefore, $x_1 = r(1 + \sin\tfrac{1}{3}\theta)$

$x_2 = r[1 + \sin\tfrac{1}{3}(\pi - \theta)]$

$x_3 = r[1 - \sin\tfrac{1}{3}(\pi + \theta)]$.

For $n=2$, x_3 is the value required.

Several contributors assumed that the loaf had no crust on bottom, in which case $x = r\left(1 - \frac{1}{\sqrt[3]{n}}\right)$.

Applied Mathematics.

171. *Proposed by W. T. Brewer, Quincy, Ill.*

A hollow copper ball is 8 inches in diameter, and in weight just sufficient to sink to its center in water. If the specific gravity of copper is 8.85, what is the thickness of copper in the ball?

Solution by W. H. Weslar, Crockett, Cal.

Let v = volume of sphere.
v' = volume of inner hollow.
d = diameter of inner hollow.
D = density of metal.

Then $v = \frac{8^3\pi}{6} = \frac{512\pi}{6}$; $v' = \frac{d^3\pi}{6}$.

$v - v'$ = volume of metal $= \frac{512\pi - d^3\pi}{6}$.

$\frac{v}{2D} =$ " " " $= \frac{512\pi}{106.2}$.

$\therefore \frac{512\pi - d^3\pi}{6} = \frac{512\pi}{106.2}$. Whence $d = 7.8464$ in.

Therefore thickness of copper $= \frac{1}{2}(8 - 7.8464) = .0768$ in.

Credit for Solutions Received.

Algebra 161. M. H. Pearson. (1)
Algebra 162. Orville Price. (1)
Geometry 163. M. H. Pearson, Orville Price. (2)
Trigonometry 165. M. H. Pearson. (1)
Algebra 166. W. T. Brewer (2 solutions), W. B. Borgers, Orville Price, G. B. M. Zerr. (5)
Algebra 167. T. M. Blakslee, W. B. Borgers, John P. Clark, H. E. Cobb, A. M. Harding (2 solutions), Orville Price, Eugene R. Smith, H. E. Trefethen (3 solutions), G. B. M. Zerr. (12)
Algebra 168. Grace E. Berry, T. M. Blakslee, W. B. Borgers, H. E. Cobb, John P. Clark, G. E. Congdon, A. M. Harding, W. F. Lady, Orville Price, Gertrude L. Roper, Eugene R. Smith, I. S. Van Gilder, G. B. M. Zerr. (13)
Geometry 169. G. B. M. Zerr. (1)
Geometry 170. T. M. Blakslee, Th. Thorson, J. H. Weaver, G. B. M. Zerr. (4)
Applied Mathematics 171. W. B. Borgers, W. T. Brewer (2 solutions), John P. Clark, Orville Price, Th. Thorson, J. H. Weaver, W. H. Weslar, G. B. M. Zerr. (9)
Total number of solutions, 49.

PROBLEMS FOR SOLUTION.

Algebra.

178. *Proposed by H. C. Whitaker, Philadelphia, Pa.*

A and B who are 500 miles apart, travel toward each other, A going at the uniform rate of 20 miles per day; B goes 12 miles the first day,

11 miles the second day, 10 miles the third day, and so on. How far from B's starting place will they meet?

179. *Selected.*

An agent gets 15% on note, and 20% and 10% on cash. Receiving goods listed at $300, he pays to the company $76.80 in cash; for how much must he give them his note on settlement?

Geometry.

180. *Proposed by Philip Fitch, Denver, Colo.*

Points P, Q are taken on the sides BC, CA of the triangle ABC so that $BP=m\cdot a$, $PC=n\cdot a$, $CQ=rb$, $QA=sb$. AP and BQ intersect at O, CO and PQ in L, CO and AB in R; find (1) ratio of AR to RB, (2) ratio of QL to LP, (3) ratio of areas of OPCQ and ABC.

181. *Proposed by W. T. Brewer, Quincy, Ill.*

The entire surface of a right cone whose slant height is 17, is equal to the surface of a certain sphere; but the volume of the sphere is twice that of the cone. Required radii of the cone and of the sphere.

Applied Mathematics.

182. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

Determine the center of gravity of a hemisphere by geometrical demonstration.

Miscellaneous.

183. *Proposed by H. C. Whitaker, Philadelphia, Pa.*

A conical wine glass, six inches deep and six inches in diameter, is full of wine. When turned 30° to one side, how much wine runs out?

SPRINGS IN CALIFORNIA.

Report in Preparation.

If Yellowstone Park, which contains so many geysers and other hot springs, is left out of consideration, California ranks as the first State in the Union in respect to the number and variety of its springs. Many of these springs find their most important use for irrigation, many others are well-known resorts, and still others, which have not been exploited, yield waters whose curative properties are probably as valuable as those of the famous springs of Germany.

In connection with its study of the underground waters of California, the United States Geological Survey has undertaken to make a systematic investigation not only of springs used as resorts and of those whose waters are bottled for table or medicinal use, but especially of those that are large enough to be the source of water for irrigation. For this purpose the southern half of the State was examined during the fall of 1908; the northern half will probably be studied during the coming summer.—*U. S. Geol. Survey.*

REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.

COMMITTEE ON INVESTIGATION: JAMES F. MILLIS, *Chairman, Francis W. Parker School, Chicago;* JOS. V. COLLINS, *State Normal School, Stevens Point, Wis.;* C. I. PALMER, *Armour Institute of Technology, Chicago;* E. FISKE ALLEN, *Teachers College, New York, N. Y.;* A. A. DODD, *Manual Training High School, Kansas City, Mo.*

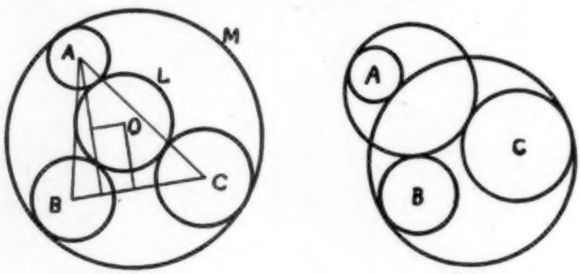
Teachers of mathematics who are interested in the movement to reform the teaching of mathematics in secondary schools by teaching the subjects more in relation to their practical uses are earnestly requested to co-operate with this committee (1) *by contributing real applied problems of algebra and geometry to be printed in these columns, where they will be accessible to all teachers for class room use,* and (2) *by using the problem material here printed in the class room with a view to determining its adaptability to the interests and needs of secondary school pupils.*

PROBLEMS.

By James F. Millis, Francis W. Parker School, Chicago.

1. To locate a gear to mesh with three given gears.

Let A, B, C, be centers of three gears with radii a, b, c, respectively,



(1) (2)

and let the distances between their centers be BC=x, CA=y, AB=z:

Required to find the formula for computing the radius of a fourth gear which will mesh with these.

If gears are all required to turn one way, there are two solutions, as in figure (1). Gear M rotates them in same direction as driving gear, and gear L in opposite direction.

The other figure suggests an arrangement for different rotations of the gears. There are eight combinations of motions possible.

By Paul E. Sabine, Worcester Academy, Worcester, Mass.

2. The boiling point of water rises about $\frac{3}{8}$°C. for each centimeter increase in barometric pressure. Given the reading of a thermometer in steam under stated pressure, to find boiling point error. For example, a certain thermometer reads 99°C. when placed in steam, the barometric pressure being 74.8 cms. Find what the thermometer would read if placed in steam under barometric pressure of 76 cms.

By Flora E. Balch, Teachers College, New York, N. Y.

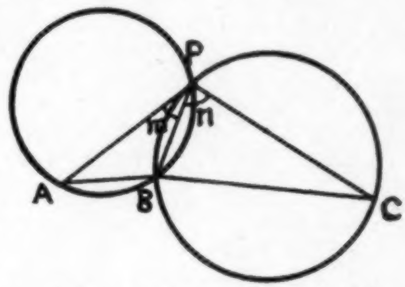
3. In a gear, $P = \frac{N \times 2}{D}$, where P is the diametral pitch, N the number of teeth, D the outside diameter. (The diametral pitch is the number of teeth in a gear per inch of pitch diameter.)

How many teeth must be cut in a wheel whose outside diameter is 5 in. in order that the diametral pitch shall be 6?

4. If two gears mesh, and S and T are the speed and number of teeth of the driving wheel and s and t the speed and number of teeth of the wheel driven, $ST=st$. If a wheel contains 50 teeth and makes 25 revolutions per minute, what number of teeth must a wheel contain that is to mesh with it and make 125 revolutions per minute?

By Jos. V. Collins, State Normal School, Stevens Point, Wis.

5. To determine the position of a point in a field by taking the backsight angles m and n on three known points, A, B, C.



6. Find for depreciation a steady percentage that will reduce a property to a fixed scrap heap value in a given number of years.

SOLUTION: Let P = cost, S = scrap value, n = number of years, and x = unknown number of per cent of annual depreciation.

Then $S = P\left(1 - \frac{x}{100}\right)^n$, whence $x = 100 - 100\left(\frac{S}{P}\right)^{\frac{1}{n}}$.

7. Country banks are required by law to keep 15% of their deposits on hand, of which 6% shall be cash in the bank and 9% may be in reserve banks. If more than 9% is in the reserve banks, the excess may be used to cancel indebtedness to other banks and in this way reduce deposit liability. It is required to find what cut on this reserve a bank can make with given conditions and still comply with the law.

SOLUTION: Let a = gross deposits, b = reserve in banks, x = possible cut in the reserve.

Then $b - x = \frac{9}{100}(a - x)$, whence $x = \frac{100b - 9a}{91}$.

NOTE.—This excess comes off the deposit amount also in counting the amount of cash on hand necessary.

8. City banks are required to keep 25% of their deposits on hands, of which half shall be cash. In this case we have

$$b - x = \frac{a - x}{8}, \text{ whence } x = \frac{8b - a}{7}.$$

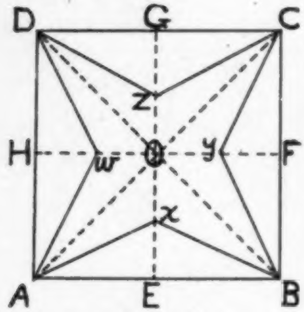
By Mabel Sykes, South Chicago High School, Chicago.

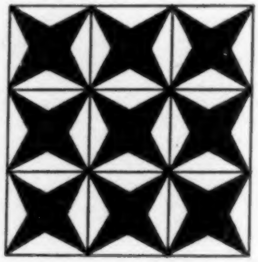
9. Carpenters and other tradesmen frequently wish to know the circumference of a circle of given radius. The accompanying graphic method is given in some of the Self-Education books as a substitute for computation:

Draw radii AO and BO at right angles. Draw chord AB and line OE perpendicular to AB, meeting circle at E and chord at D. To 6 times the radius add the line DE. The resulting line is approximately the length of the circumference.

Compute the approximate per cent of error, using $\pi=3.14159$. Answer, about .15%.

10. These figures show a parquet floor design and one of the units of the design enlarged.

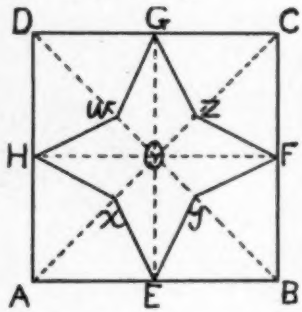


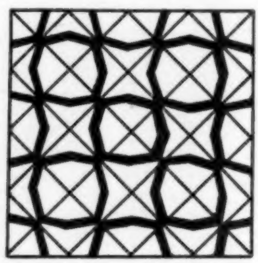


If ABCD is a square and x, y, z, and w are the middle points of the semidiameters OE, OF, OG, OH, respectively, prove that the area of the star AxByC— is one-half of the area of the square ABCD.

Suggestion: Compare triangles OBx and OBE. Triangles having equal altitudes are to each other as their bases.

11. These figures show another parquet floor design and one of the units of the design enlarged.

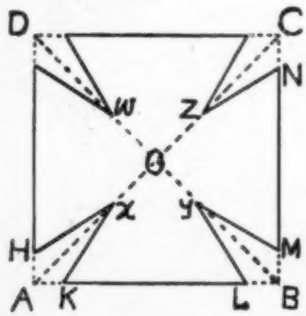


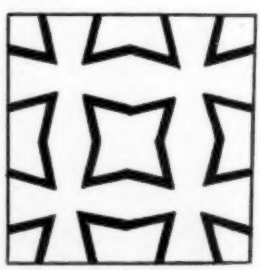


(a) If ABCD is a square and $Ox=Oy=Oz=Ow=\frac{1}{3}$ of the semidiagonal, prove that the area of the star EyFzG —is one-third of the area of the square ABCD.

(b) How must the figure be constructed in order that the octagons enclosed by the stars shall be regular?

12. These figures show a parquet floor design and one of the units of the design enlarged.





(a) ABCD is a square. If AK=LB=BM=$1\frac{1}{2}$, Ox=Oy=Oz=Ow=2, and AB=12, find the area of the quadrilateral AHxK and of the cross KLyMNz—.

(b) If AB=12, AK=$1\frac{1}{2}$ and Ox=$\frac{1}{4}$OA, find the area of the quadrilateral and of the cross.

(c) If $AB = a$, $AK = \frac{a}{n}$ and $Ox = \frac{AO}{k}$, find the areas mentioned in the preceding questions.

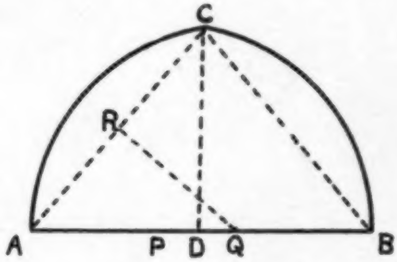
(d) Study the case when $AK = \frac{a}{2}$.

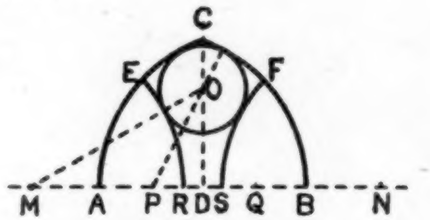
13. The figure is the "drop arch" seen in many Gothic buildings.

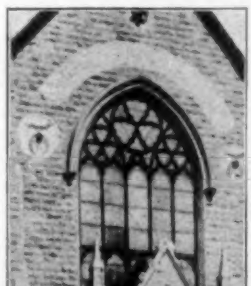
(a) Given AB and CD. CD is the perpendicular bisector of AB. Construct the arcs AC and CB with centers on the line AB.

(b) Prove that triangles ACD and ARQ are similar, and hence show that $AQ = \frac{AD^2 + CD^2}{2AD}$.

(c) If AB=12 and CD=9, find AQ.







14. This figure is from a photograph of the window in the front of Medina Temple, Chicago, accompanied by a diagram showing the construction of the central circle.

(a) If circle O is tangent to arcs CA and CB, or to arcs ER and FS, prove that its center must be on CD, the perpendicular from C to AB.

(b) If AQ=BP=$\frac{3}{4}$AB, AR=SB=$\frac{2}{5}$AB, MR=SN=AQ, and AB=s, show that the radius of circle O is $\frac{11s}{50}$.

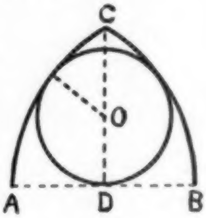
Suggestion: In triangles OMD and OPD, $OM^2 - MD^2 = OP^2 - PD^2$, if the radius of circle O is r, we get the equation

$$\left(\frac{3s}{4} + r\right)^2 - \left(\frac{17s}{20}\right)^2 = \left(\frac{3s}{4} - r\right)^2 - \left(\frac{s}{4}\right)^2.$$

(c) If AB=s and AQ=BP=MR=SN=kAB=ks, show that the

radius of circle O is $\frac{3s}{50}\left(\frac{5k-1}{k}\right)$.

NOTE.—By the use of (b), Problem 13, and (c) Problem 14, the radius of circle O can be found for any window of this type, if the span and altitude are known.



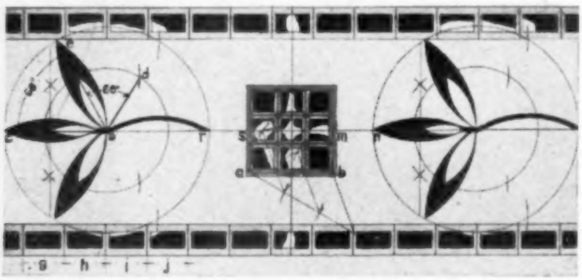
15. Inscribe the circle in the figure ABC, tangent to the arcs AC and CB and to the line AB, by means of the problem: To construct a circle tangent to a given circle and to a given line at a given point.

NOTE.—This is a more elegant solution of the problem than that given in SCHOOL SCIENCE AND MATHEMATICS, May, 1906, and reprinted June, 1909.

By R. S. Howlett, North Side High School, Denver, Colorado.

16. A geometrical embroidery pattern.—*St. Gaul, Switzerland.*

In the construction, angle cbe=60°; angle cbd=60°; ab= side of



square; $rs=mn$; ab is divided into three equal parts; and spaces g, h, i, j, etc., are equal. In the drawing, open spaces in linen and solid embroidery are in black, and construction lines are all full black lines.

NOTE.—The last problem represents a field of application of elementary geometry that needs to be thoroughly investigated for problem material. It is in such fields as this that problems should be found that are of most interest to girls.

Mathematics Teachers in Private Schools are asked to give their aid to the investigation of mathematics teaching now being conducted by the International Commission on the Teaching of Mathematics. The Committee on Private Secondary Schools is about to distribute a questionnaire which, if generally answered, will give, in connection with the data collected by other committees, a comprehensive view of the condition of mathematics teaching in this country. The reports which are to be published are sure to be of great value to all teachers of the subject. Let everyone see that the blanks are filled out promptly and accurately for his own school, and lend his support to the movement whenever he observes an indifferent attitude on the part of others.

WILLIAM E. STARK,
Chairman Committee No. IV.

OPAQUE PROJECTION.

Lantern slide, scientific and technical projection have reached so high a point of perfection that it would seem impossible to suggest new kinds of apparatus which might be added to it. This is very largely the view taken by the most progressive manufacturers, and is coincided in by the McIntosh Stereopticon Co. of Chicago, which has for a number of years sustained a high reputation for high grade instruments. They have for some time directed all the attention of their experts to improving present models rather than to inventing new machines.

Following closely upon the popular craze for souvenir post cards, a number of instruments have been marketed for projecting cards and pictures. These have been sold at prices ranging from $5.00 to $15.00, and while they are complete instruments they are nothing more or less than toys. They are provided with cheap lenses, cheap illumination, and can only be used for entertaining and amusing children. While they are very amusing, scientific toys, and will project objects such as pictures, coins, etc., they cannot be used to produce a picture more than a very few feet in diameter, and are altogether useless for scientific purposes. On the other hand, the high grade projectors suitable for laboratory and lecture work have been heretofore so expensive as to be out of the reach of the average user and the smaller schools and colleges.

A new instrument at a comparatively low price has recently been perfected, however, in the McIntosh Opaque Projector which is equipped with a high grade Bausch and Lomb lens, and is arranged to give a distinct, strong, flat image of any object which is placed in it for projection, such as Perry pictures, diagrams, prints, pages from a book or in fact almost any opaque object. It has a book holder of an entirely new style, and is provided with an arc lamp and rheostat to furnish about 5,000 candle power illumination.

The great difficulty with opaque projectors in the past has been the loss of light due to an arrangement of mirrors which has hitherto been impossible to avoid. This instrument is so arranged that a picture can be shown on the screen by direct reflection, thus adding to its brilliancy and clearness.

It is possible, also, where desired, to add an Optical Front to this projector in such a manner that lantern slides can be used interchangeably with opaque objects. The McIntosh Stereopticon Company will be glad to send their complete descriptive circular to anyone interested, upon demand.

Teachers of botany, zoölogy, and of agriculture as well as those generally interested will find some valuable suggestions in a recent report (Farmers Bulletin 370, U. S. Dept. of Agr.) entitled, "Replanting a Farm for Profit." This is a 36 page pamphlet full of the results of practical experiments and of suggestions for those who are concerned with making modern scientific knowledge bear fruit in practical affairs.

O. W. C.

BEGINNING OF FORESTRY IN THE UNITED STATES.

When did the United States begin the practice of forestry? Few persons can answer this question correctly. Most people are of the opinion that the beginning of forestry in this country was of very recent origin, and that the first step in that direction was taken among the mountains of the far West. Neither fact is correct.

While Washington was serving his first term as President of the United States, a recommendation came to him that the government ought to buy live oak islands on the coast of Georgia to make sure of a supply of ship timber for war vessels. The idea appears to have originated with Joshua Humphreys, whose official title was "Constructor of the United States Navy," although about the only navy then existing was made up of six ships on paper, and not one stick of timber to build them had yet been cut. The vessels were designed to fight the north African pirates.

Five years after the recommendation was made Congress appropriated money to buy live oak land. Grover and Blackbeard Islands on the coast of Georgia were bought for $22,500. They contained 1,950 acres.

Louisiana was bought soon after, and in 1817 the Six Islands, of 19,000 acres, and containing 37,000 live oak trees, were withdrawn from sale, and set apart as a reserve. In 1825, Congress appropriated $10,000 to buy additional live oak land on Santa Rosa Sound, western Florida, and subsequently other Florida timber lands, aggregating 208,224 acres, were reserved.

Up to that time nothing more had been done than to buy or reserve land for the timber growing naturally upon it; but the work was to be carried further upon the Santa Rosa purchase. The plan included planting, protecting, cultivating, and cutting live oak for the navy. That timber was then considered indispensable in building war vessels. Much had been said and written of the danger of exhaustion of supply. Settlers destroyed the timber to clear land, and European nations were buying large quantities for their navies. In response to repeated warnings the government finally took steps to grow timber for its own use.

Young oaks were planted on the Santa Rosa lands. Difficulty was experienced in inducing young trees to grow. The successful transplanting of the oak is not easy, unless done at the proper time and in the right way. The plantations at Santa Rosa were generally unsuccessful; but large quantities of acorns were planted, and a fair proportion of them grew. But the chief efforts were directed to pruning, training, and caring for the wild trees. Thickets about them were cut away to let in air and light.

What the ultimate success of the forestry work would have been cannot be told. The Civil War brought a complete change in war vessels by substituting iron for wood. Forestry work stopped. The timber reserves were neglected. Squatters occupied the land. After a number of years all the reserves, except some of the Florida land, were opened to settlement.—*U. S. Geol. Survey.*

PERSONALS.

W. E. Durstine of the Joliet, Ill., High School goes to the Technical High School, Cleveland, Ohio, in charge of the department of geography.

Ernest Roller of Lafayette, Ind., will teach physics in the Michigan Agricultural College this year.

Howard W. Adams, head of the department of science in the Elgin High School, Ill., has accepted the position of professor of chemistry at the Illinois State Normal University, Normal, Ill.

J. E. Weyant goes from Calumet, Mich., to the physics department of the Shortridge High School, Indianapolis, Ind.

BOOKS RECEIVED.

Leitfaden für Physikalische Schülerübungen von Hermann Hahn, mit 225 Text figuren. Pp. 354. 1909. Berin Verlag von Julius Springer. Price, 3 marks.

Primer of Sanitation. Being a single work on Disease Germs and How to Fight them. By John W. Ritchie. Pp. 200. 1909. World Book Co., Yonkers, N. Y.

First Year in Chemistry. A text in Elementary Chemistry for Secondary Schools. By Wilhelm Segerblom. Pp. 410. 1909. Exeter, N. H. Exeter Book Publishing Co.

The Nature Study Idea. An Interpretation of the New School Movement to put the Young into Relation and Sympathy with Nature. By L. H. Bailey. 3rd edition. Revised. Pp. 246. 1909. New York: The Macmillan Co. Price, $1.25 net.

The Teaching of Arithmetic. By David Eugene Smith. Pp. 120. 1909. New York. Teachers' College, Columbia University.

Exercises in Geometry. By Grace Lawrence Edgeth. Pp. 81. 1909. Boston. D. C. Heath & Co.

Proceedings of the Sixty-third Annual Meeting of the New York State Teachers' Association. Pp. 414. 1909. Albany, N. Y. University of the State of New York.

Exercises in French Composition for Schools and Colleges. By William Koren. Pp. 237. 1909. New York: Henry Holt & Co. Price, 75 cents.

Laboratory Manual of First Year Science for Secondary Schools. By W. S. C. Russell and H. C. Kelly. Pp. 163. 1909. New York: Henry Holt & Co. Price, 75 cents.

New England Association of Chemistry Teachers. Report of the Thirty-fifth Meeting. Pp. 23.

Treatise on Qualitative Chemical Analysis. By J. T. Sellers, Mercer University, Macon, Ga. 12 mo. cloth. 176 pages. Mailing price, $1.05. Ginn & Co., Boston.

Manual of Qualitative Chemical Analysis. By J. F. McGregory, Colgate University. Cloth, xiv+133 pages. Mailing price, $1.10. Ginn & Co., Boston.

A Treatise on Differential Geometry. By Luther Pfahler Eisenhart, Princeton University. 8 vo. cloth. 474 pages, with diagrams. Mailing price, $4.70. Ginn & Co., Boston.

BOOK REVIEWS.

The Teaching of Arithmetic, by Alva W. Stamper, Ph.D., Head of the Department of Mathematics, State Normal School, Chico, Calif. Pp. 80. 50 cents. 1909. Normal Junior Press.

Lesson Plans in Arithmetic. Pp. 17. 20 cents.

The author prepared these pamphlets for his classes in the normal school with the aim of giving them a view of the larger problems of teaching arithmetic, suggestions as to methods, and a review of subject matter of arithmetic. Though the lists of reference books include only those accessible to the students of the school, and though under some of the topics the work is merely outlined, there are many valuable suggestions in the pamphlets for all teachers of arithmetic.

H. E. C.

Coördinate Geometry, by Henry Burchard Fine and Henry Dallas Thompson, Princeton University. 309 pages. Price, $1.60. 1909. The Macmillan Company.

The fact that this book is the work of the authors themselves, that it has received their closest personal attention in all details, and that it has been used for three years in a preliminary edition so that necessary changes could be made in it, is a sufficient guaranty of its excellence.

In the second chapter the five forms of the equation of a straight line are derived and grouped in such a way that the student can remember them. Oblique axes are used in this chapter alone. Only forty pages precede the conics, and the suggestion of the authors that the parabola be studied immediately after the straight line seems to be worth following. The usual constant ratio definition is given for the conics; and the discussion of a more general form of the equation of each of the conics with good figures comes at the right time to make it possible for the student to become thoroughly acquainted with the various forms of the equations and to learn to sketch the curve quickly.

A choice of three methods is given for deriving the equation of the tangent to each of the conics at a point on the curve. The third method in each case is the method of the calculus carefully disguised. It seems exceedingly unfortunate that in this country authors and teachers should cling so tenaciously to traditions of the past. Why should not the process of differentiation be defined and illustrated, and used to obtain the equation of the tangent?

In this book three pages are given to finding the equation of the tangent of the parabola, three pages for the ellipse, and three pages for the hyperbola. The work for the last two curves is simply a repetition of that for the parabola almost word for word, with nothing to indicate the similarity in method. These nine pages would be sufficient for a thorough discussion of the dy/dx method, and the student would then have at hand a general method of finding a tangent to a curve. Our authors and teachers should follow the lead of the German schools, if they must be followers rather than leaders.

There are many things to commend in this book, and it will no doubt be used in many colleges; it is for the pure mathematician rather than for the technical student. The figures are well drawn, and the type, paper, and binding are of the best. H. E. C.

Wood Turning, by George Alexander Ross, Instructor in Charge of Wood Turning and Pattern Shops, Lewis Institute, Chicago. 12mo. Cloth. Illustrated., 76 pages. List price, $1.00; mailing price, $1.05. Ginn and Company, Chicago.

This is a splendid book for class work in wood turning, and one with any knack for tools ought to be able to become expert in this art without other help after a few trials with chisel and lathe. The book covers a wide range of work which is helpful alike to teacher and pupil. The first few pages are devoted to a brief discussion of the development of the lathe and its parts. The tools used in turning and how to sharpen them is told. Several exercises are given beginning with the simple cylinder, increasing in hardness until all phases of wood turning are covered. The appendix gives directions how to stain and finish work in the lathe, also recipes for making and applying stains. The mechanical work in the book is perfect, the drawings and cuts being well executed. C. H. S.

College Algebra, by H. L. Rietz, Ph.D., Assistant Professor of Mathematics, University of Illinois, and A. R. Crathorne, Ph.D., Associate in Mathematics, University of Illinois. 261 pages. Price, $1.40. 1909. Henry Holt & Co.

It would seem as if this book would be widely used in technical schools and colleges where emphasis is laid on the simple and direct treatment of those topics which are really needed in engineering work, and where the attempt is made to establish close relations between mathematics and the work in the laboratories and shops.

In the first hundred pages there is a review of the high school algebra, but it is more than a review since the subject-matter is seen from new viewpoints, and there are many excellent practical problems from the fields of physics and engineering. Some of the topics usually included in a college algebra course are wisely omitted, and the material is restricted to the development of those central points which have been found essential. The discussion of the theory of equations is illuminated by continual reference to the graph. Especially is Horner's method made easy of comprehension to the student by graphical methods. However, it is a question if it would not be well to omit Horner's method entirely, and find irrational as well as rational roots by graphical methods. In general, the roots are needed only to three or four significant figures, and Horner's method is easily forgotten.

The authors have attempted no critical study of fundamentals, but have indicated in the discussions just what is assumed and what is proved. They have written a book which will give a student not only training in mathematical thinking and expression but also a working knowledge of the mathematical principles and methods required in elementary physics and engineering problems. H. E. C.

The Pupils' Arithmetic, Primary Book, Part One. By James C. Byrnes, Member of Board of Examiners, Department of Education, New York; and Julia Richman, District Superintendent of Schools, New York; and John S. Roberts, Principal of Public School, 62 Manhattan, New York. 216 pages. Price, 26 cents. 1909. The Macmillan Company.

The work of the first three years is covered in this book. It is arranged on the topical plan, since the authors think that the spiral plan while entirely sound in principle and practicable in operation tends to a lack of definiteness in teaching. There is a large number of problems in applied arithmetic, but there is a decided lack of problems which require the pupil to estimate, measure, draw, and construct.

H. E. C.

Elements of Physics, by Henry Crew, Northwestern University, and Franklin T. Jones, University School, Cleveland. xi+435 pages. 357 cuts. The Macmillan Company, New York.

Until quite recently the secondary school text-books on Physics too frequently were abridged editions of some college manual based on the encyclopedic idea. Continuity of thought was absent and each succeeding section heralded new ideas having but distant apparent relations to those previously presented. Revision meant but two things: the additional description of newly devised mechanisms and the alteration of theories that proved no longer tenable.

The better recent books are an attempt at a more logical presentation of the elementary principles with special emphasis on continuity of order in their development.

Enthusiastic authors and those not intimately acquainted with the psychology of students of a high school age commonly make the mistake of presenting more subject matter than our classes can assimilate. Many of our text-books have consequently become somewhat extended and intensive, requiring certain omissions and not infrequent explanation of what the text meant to state. Books of this nature are easy to write. Great skill, however, is required in presenting the more essential facts in a thoroughly logical and continuous form, retaining the basal facts, omitting what is essentially collegiate work and still covering sufficient ground to fill an entire school year without undue repetition in the nature of reviews.

Crew and Jones's "Elements of Physics" is a book of the latter type. It is a complete and radical revision of Professor Crew's former book. The work of revision has been most thoroughly performed and with great success.

The development of the more essential equations in mechanics is simple and effective. It is plainly shown that these equations are not a piece of mathematical jugglery invented for the confusion of the illogical mind, but solely a shorthand method of expressing physical relations for the purpose of simplifying their applications.

The chapter preceding sound is one on wave motion. This is a very desirable innovation and correlates the chapter on mechanics with those on sound and light and also to a lesser extent with those

on heat and electricity. The chapter on sound is brief and omits much that is non-essential.

The fundamental facts of heat and electricity are well presented, though little space is devoted to their practical applications. The authors apparently do not believe in making an elementary text-book on physics serve as an engineer's manual. This is a good idea, for too frequently valuable time is lost explaining applications when the underlying fundamental principles are but half understood.

In the last chapter the fact that light is a type of wave motion is well presented.

Next follows an appendix of problems covering the entire book. These problems were selected from many examinations and are well chosen.

The book is without doubt one of the best text-books on elementary Physics. It merits the careful reading of every secondary school-teacher and should meet with a wide adoption. E. J. R.

Webster's New International Dictionary of the English Language. Pp. 2,700. G. & C. Merriam Co., Springfield, Mass. 1910.

The announcement of a new dictionary marks an epoch in the growth and development of a nation. Since Webster sent out a dictionary in New England in 1828, four general revisions have been made: the first in 1840; second, 1847; the Unabridged in 1864, and the International in 1890. For the past six years a large staff of editors, under the supervision of the late Dr. William T. Harris, has been at work on the revision. In this new edition several radical changes have been made. The separate vocabularies at the end of the volume have been distributed throughout the entire work, with the exception of the pronouncing glossary, the biographical dictionary and the arbitrary signs used in printing and writing.

The most noticeable change from the International is the novel arrangement of the pages. Each page is divided into two portions; the lower in smaller type and narrower columns than the upper. The lower section contains the foreign phrases, abbreviations, obsolete words, uncommon dialect words, scientific terms that are of rare occurrence, and in general, words not in common use; the upper portion containing the general vocabulary. This relieves the main vocabulary of many confusing entries and saves space by making the minor entries more accessible by a more compact arrangement in narrower columns.

A second space saving feature is the giving of a systematic definition to certain general words and by giving information in tabulated forms. For example: *army organization, coin, era, star*, etc. A third means for saving space is the defining of many formal derivatives by reference to their prefixes or suffixes. For example: *ness*, and *ish*.

The editors had four guiding principles in their work: first, they aim to preserve the exactness and comprehensiveness, which have always characterized the Webster definitions, and to reinforce those qualities by a more historic order and fuller presentation. For example: the word *but* is given one hundred twenty lines instead of sixty-five in the old International; eleven definitions as against six, and

thirty-three quotations instead of fourteen. Within the last twenty years the English language has been enriched by reason of the natural expansion, growth and development of the sciences and mechanic arts, so that the number of words and phrases is much greater in number than in the previous editions. It is thoroughly up-to-date, containing many words not found in any other dictionary. Such slang phrases as *bluff*, *stand-pat*, *stand-patter*, *cinch*, *rough-house*, *graft* and *make good* are given. According to the publisher's preface, more than 400,000 words and phrases are defined.

The dictionary contains much matter of an encyclopedic character. For example: under *automobile*, two pages of cuts with one page of descriptive matter is given. Two columns with six cuts are given to *electric*. One-half column is given to *ion;* three columns to *dye*, with a table including all the common dyes, with the mechanical names, color, kinds of goods, uses, etc.

The page is divided into two portions, as mentioned above, to increase the ease and convenience in finding words.

In spelling the common usage is accepted. The changes prescribed by the Simplified Spelling Board have not been indorsed, with the exception of a few common words like *program*. As past participles in *t* instead of *ed* are merely revivals of obsolete or archaic forms, they are placed in the lower section and so marked.

The synonyms constitute practically a new feature of the work. As compared with the old International, the number of articles on synonyms, according to the preface, has been increased from 600 to 1,448; separate words treated, from 1,687 to 4,810; of quotations, from 400 to about 6,000. The New International gives a great deal of space to early English dialect forms, so that students of English literature can pursue their studies without consulting any other glossary.

The old Webster system, the best and most widely known system for indicating pronunciation, has been retained with a few alterations. The old, familiar stand-bys, such as lists of authors quoted, brief history of the English language, guide to pronunciation, and seventy pages of classified pictures are also to be found. As to what advantage the arrangement of these illustrations in one place affords, the writer is unable to determine. The space could be much better devoted to other matters, or the book could be made that much smaller to advantage. This new dictionary is as much superior to the Old International as that was to the Unabridged. C. M. T.

The Symposium on the Teaching of Chemistry in Secondary Schools, which has been running in the last four issues, will be continued in the January number with articles by Dr. John C. Hessler and Professor Wilhelm Segerblom.

For lack of space the page on "Articles from Current Magazines" is omitted from this issue.

Remember the meeting of the Central Association of Science and Mathematics Teachers at the University of Chicago, Nov. 25-27. Take advantage of the reduced rates on the railroads and attend conference.

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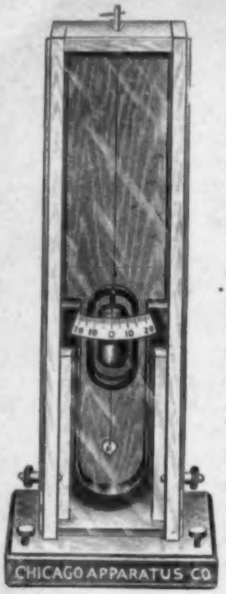

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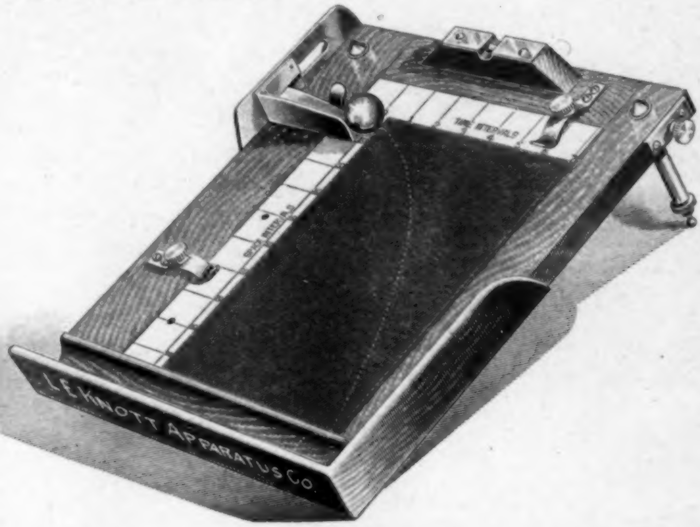

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